※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

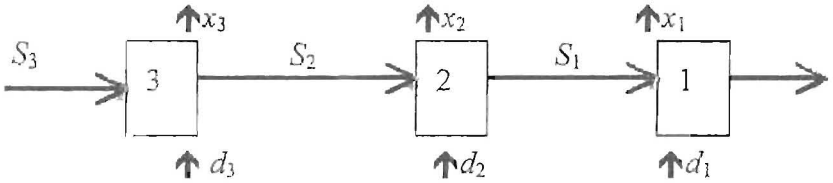
1. (20%) Three products A, B, C, with per-unit profit margin \$60, \$100, \$50, are to be assembled in two production lines. Processing time of different products and weekly capacity of the production lines are listed below:

Processing time (hours)	Product A	Product B	Product C	Capacity (hours)
Production line 1	4	6	3	2000
Production line 2	5	7	4	2400

Demands of three products are forecasted as below: between 50 and 100 (A), between 150 and 200 (B), between 100 and 150 (C). The fixed cost and time to setup the production lines are \$1000 and 24 hours for production line 1, and \$800 and 18 hours for production line 2. Formulate this production planning problem as a mathematical programming model. Clearly define decision variables, objective function and constraints, to determine the allocation of production lines and the optimal production quantities, in order to maximize the total profit.

2 (15%) For the following dynamic programming model:

$$\text{Maximize: } \sum_{i=1}^{3} x_i \text{ , Subject to: } \begin{cases} S_{i-1} = 2S_i - d_i, & i = 2,3 \\ 0 \le d_i \le S_i, & i = 1,2,3 \\ x_1 = 2d_1, \quad x_2 = d_2, \quad x_3 = d_3^2 \end{cases}$$



What are the stages and states in the above model? Solve the above model by dynamic programming for S_3=6, give the optimal policy and the optimal objective value.

3. (15%) Use either the Bisection method or the Newton's method (Choose only one) to solve the following non-linear programming problem: {Maximize $f(x) = 12x - 3x^4 - 2x^6$, $0 \le x \le 2$. }. Use starting point x=1 for Newton's method. Execute 5 iterations and tabulate the computational result. Give the optimal x value and the objective value.

(背面仍有題目,請繼續作答)

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4. An important unit consists of two components placed in parallel. Only one component is operated at a time, but both components are kept operational as often as possible by repairing them as needed. An operating component breaks down in a given period with probability 0.3. When this occurs, the parallel component takes over, if it is operational, at the beginning of the next period. The repair of a component starts at the beginning of first available period and is completed at the end of the next period.

   (a) Formulate the condition of this unit as a Markov Chain by defining its states space and giving its one-step transition matrix. (10%)

   (b) What is the probability that the unit will be operable after 2 periods? (5%)

   (c) If it costs $ 50000 per period when the unit is inoperable and zero otherwise, what is the expected average cost per period? (10%)

5. A surveillance system requires working radar units to scan the horizon. When three or more units are up, the system is said to be in the "operational" mode. If only two are working, it is in the"reduced effectiveness" mode. When fewer than two are working, the system is in the "failed" mode. Four radar units have been installed. Each unit fails randomly and at the same average rate - once every 400 hours. When a failure occurs, the unit is sent to a repair shop where one unit is worked at a time. If more than one unit has failed, the others must wait in a queue. The repair operation takes an average of 50 hours, but the actual repair time has an exponential distribution. Compute the proportion of time that the system will be in each of the three modes.(15%)

6. Consider a queue system with c servers. Customers arrive randomly with a rate λ and the service time is exponential with rate μ. Derive the probability that no customer in the system.(10%)