※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

一. Two fair dice are rolled. Let X be the absolute difference between the outcomes of the dice. For example, if the outcomes are 2 and 6, then X=4.

a. (5%) Find the p.m.f. and c.d.f. of X.

b. (5%) Find $P(0 < X \leq 3)$ and $P(1 \leq X < 3)$.

二. (10%) Consider a random sample $X_1, \ldots, X_n$ from the pdf

$$f(x;\theta) = 0.5(1+\theta x) \qquad -1 \leq x \leq 1,$$

where $-1 \leq \theta \leq 1$ (this distribution arises in particle physics). Show that $\hat{\theta} = 3\bar{X}$ is an unbiased estimator of θ.

三. (10%) John and Mary have agreed to meet for lunch between noon (0:00 P.M.) and 1:00 P.M. Denote John's arrival time by X, Mary's by Y, and suppose X and Y are independent with pdf's

$$f_X(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 2y & 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

What is the expected amount of time that the one who arrives first must wait for the other person?

四. The desired percentage of SiO_2 in a certain type of aluminous cement is 5.5. To test whether the true average percentage is 5.5 for a particular production facility, 16 independently obtained samples are analyzed. Suppose that the percentage of SiO_2 in a sample is normally distributed with σ=0.3 and that $\bar{x} = 5.25$.

a. (10%) Does this indicate conclusively that the true average percentage differs from 5.5? Use $\alpha = 0.01$

b. (5%) If the true average percentage is $\mu = 5.6$ and a level $\alpha = 0.01$ test based on n=16 is used, what is the probability of detecting this departure from H_0?

c. (5%) What value of n is required to satisfy $\alpha = 0.01$ and $\beta(5.6) = 0.01$?

(背面仍有題目,請繼續作答)

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五、Brakes Shop, Inc., is a franchise that specializes in repairing brake systems of automobiles. The company purchases brake shoes from a national supplier. Currently, lots of 1,000 brake shoes are purchased, and each shoe is inspected before being installed on an automobile. The company has decided instead of 100% inspection to adopt an acceptance sampling plan.

1. (4%) Explain what is meant by the acceptance sampling plan, and state its null hypothesis.
2. (8%) The manager of the quality control department of the company is willing to tolerate both 0.10 probability of rejecting a lot with 20 defective items and 0.20 probability of accepting a lot with 150 defective items. Give an acceptance sampling plan that can meet the requirements, and justify your answer.

六、Let X and Y be two random variables defined on sample space S = {1, 2, 3, 4, 5, 6}. The samples collected for these two random variables are shown in the following table.

	1	2	3	4	5	6
X	20	12	25	10	18	15
Y	10	10	20	10	15	15

1. (4%) Give the graphical representation of the probability distribution for X estimated from its sample.
2. (12%) The goodness-of-fit test is employed to test whether it is appropriate to assume that X and Y follow the same probability distribution. Formulate the null hypothesis and calculate the test statistic.
3. (4%) Provide all possible rules for rejecting the null hypothesis at significance level α.

七、The marketing department of a company has designed three different boxes for its product. It wants to determine which box will produce the largest amount of sales. Each box will be test marketed in five different stores for a period of a month. Below you are given the information on sales.

	Store 1	Store 2	Store 3	Store 4	Store 5
Box 1	210	230	190	180	190
Box 2	195	170	200	190	193
Box 3	295	275	290	275	265

1. (4%) State the null and alternative hypotheses.
2. (8%) Construct an ANOVA table.
3. (6%) What are necessary assumptions for this hypothesis testing?

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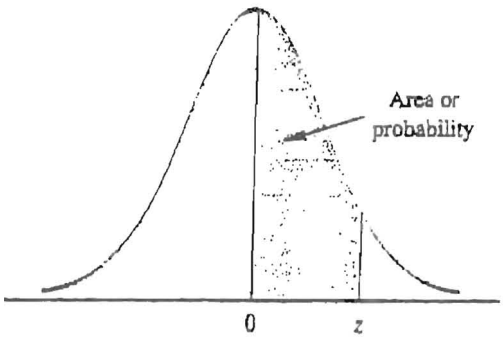
STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990