※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Four machines are located at the following x_1 and x_2 coordinates: (0, 0), (10, 0), (0, 15), and (20, 20). Let the coordinates of the new machine be (x_1, x_2). The material flows from the new machine to the four machines are 4, 1, 3, and 2 respectively. Formulate the problem of finding the optimal location of the new machine as a linear programming model to minimize the sum of material flow cost, i.e., the amount of flow multiplied by the distance between two machines, where rectilinear distance is used in the layout. Clearly define decision variables, objective function and constraints, to determine the location of new machine, in order to minimize the total material flow cost. (Do not solve this LP problem.)

2. (15%) Consider a project consisting of nine activities (A, B, . . . , I) with the following precedence relations and time estimates:

Activity	A	B	C	D	E	F	G	H	I
Predecessor	-	-	A, B	A, B	B	D, E	C, F	D, E	G, H
Time (Days)	15	10	10	10	5	5	20	10	15

Draw the project network designating the activities by arcs and event by nodes. Determine a project schedule listing the earliest and latest starting time, and slacks for each activity. Formulate this problem as a linear programming model to find the critical path. (Do not solve this LP problem.)

3. (20%) Orders from customer and production-inventory costs for the next three weeks are listed below

Week i	Order quantity d_i	Setup cost for production (\$/lot) k_i	Inventory carrying cost (\$/unit) h_i
1	3	3	1
2	2	7	3
3	2	6	2

The initial inventory at the first week is 1. In addition to the fixed setup cost, the production cost to manufacture y_i units, i=1, 2, 3 is:

$$c_i(y_i) = \begin{cases} 10y_i & 0 \le y_i \le 3 \\ 30 + 20(y_i - 3) & y_i \ge 4 \end{cases}$$

No shortage is allowed and the inventory carrying cost at one week is based on the ending inventory at that week. Formulate the above production planning problem as dynamic program to minimize the total production and inventory costs. What are the stages and states in the above model? Solve the above model by dynamic programming; and give the optimal production policy and the optimal objective value.

（背面仍有題目，請繼續作答）

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4 (25%) Customer accounts receivable at NCKU Company are split into the following classifications each month (30 days): 1 = Current (0-30 days past due); 2 = 31-60 days past due; 3 = 61-90 days past due; 4 = more than 90 days past due

A customer in state 1 will pay his/her bill in the next month with probability 0.9.
A customer in state 2 will pay his/her bill in the next month with probability 0.5.
A customer in state 3 will pay his/her bill in the next month with probability 0.3.
A customer in state 4 will pay his/her bill in the next month with probability 0.2.

If a customer enters state 4, creditors will be notified and sent after the customer. Assume that the customer is currently in state 2. Notice that paying bill means we go to state 1 (account is no longer overdue).
The transition matrix is as follows:

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0.3 & 0 & 0 & 0.7 \\ 0.2 & 0 & 0 & 0.8 \end{bmatrix} \end{array}$$

a) (5 points) What is the probability that he pays his bill before the company sends its creditors after him?
b) (5 points) What is the probability that he finally pays his bill in week 3?
c) (5 points) What is the probability that he pays his bill by week 3?
d) (5 points) In the long-run, what percent of the time will the creditors be after him?
e) (5 points) Again suppose we start at state 2 and that we would like to simulate what happens over the next 6 months. Suppose we generate the following 6 random numbers (in order): 0.58, 0.42, 0.64, 0.05, 0.84, 0.14. Which state will the account be in six months from now?

Hint: You may use the following result about inverses of 2 x 2 matrices for these problems:

$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

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5 (25%) NCKU Company owns a small lemonade stand in Tainan. The stand is operated by one salesperson. Customers will only visit the lemonade stand if there are no other customers there. Otherwise they leave and come back later. The company has hired a consultant to observe the lemonade stand and to study its behavior. The consultant has observed that both interarrival and service times follow an exponential distribution. Also, during 10,000 minutes of observation, it was observed that 20,000 customers received service and 4,000 actual minutes of service were dispensed.

a) (4 points) In our queueing notation, what type of queueing model is this?
b) (18 points) Based on the consultant's observations, estimate as many of the following quantities as you can for this system: λ, μ, each of the π_j (steady-state probabilities), λ_{eff} (effective arrival rate), L, L_q, W, and W_q.
c) (3 points) Estimate the number of lost customers during the 10,000 minutes that were observed.