

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider the following problem

$$\begin{aligned} \text{Minimize } Z &= 6x_1 - 3x_2 + 10x_3 \\ \text{subject to } & x_1 - x_2 + 2x_3 \geq 4 \\ & 2x_1 + x_2 + x_3 \geq 1 \\ & x_2, x_3 \geq 0. \end{aligned}$$

(a) Write its dual (5%)

(b) Solve both the primal and dual problems (10%)

2. A company owns two plants that manufacture a basic chemical compound for two customers at the rate of 660 and 800 tons per month. The monthly production capacity of plant 1 is between 400 and 800 tons and that of plant 2 is between 450 and 900 tons. The production cost per ton in plants 1 and 2 are \$25 and \$28, respectively. The transportation costs per ton from plant 1 to clients 1 and 2 are \$3 and \$4, and from plant 2 costs are \$5 and \$2, respectively.

(a) Formulate the problem as a network model to minimize the total cost. (10%)

(b) Give a feasible solution of the problem. Is your solution optimal? Why is it optimal or not? (10%)

3. Use the KKT conditions to find an optimal solution to the following NLP:(15%)

$$\begin{aligned} \max z &= -x_1^2 - x_2^2 + 6x_1 + 12x_2 \\ \text{s.t. } & x_1 + x_2 \leq 6 \\ & x_1 \leq 3 \\ & x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

4. Your boss asked your colleague to study the behavior of a quick car-wash store. Your colleague collected the inter-arrival times, service times, and waiting times in that store of 1000 customers, and reported to your boss that, on average, customers arrived at that store every forty minutes, each server used thirty minutes to wash a car, and every customer spent fifty minutes in that store. Your boss asked your colleague about the average waiting time in queue, average number of customers in the store, and average number of customers in queue. He said he did not collect these data. Can you answer these questions for your colleague, without re-collecting the data? (10%)
5. Customers arrive at a service station with exponentially distributed inter-arrival times of mean five minutes. The service times of the server are also exponentially distributed, with a mean of four minutes. After the service, 50% of the customers leave the system, while 50% are not satisfied, and ask another server, with a mean service time of six minutes, to serve again. After the second service the customer will be satisfied, and leave the system. Calculate the expected number of customers you will find in the store and the expected time that a customer will spend in the system. (15%)
6. Suppose the demand for a product in a unit of time is probabilistic, with a mean of D . The lead time demand x has a density function $f(x)$ and distribution function $F(x)$. Furthermore, shortages are allowed, and are backordered. Let k be the set up cost for each order, p be the shortage cost and h be the holding cost per unit per unit time. Formulate the approximate total relevant cost per unit time, including (1) ordering cost, (2) shortage cost, and (3) holding cost, for ordering Q units each time when the inventory drops down to the reorder point R , and derive the equations for solving the optimal Q^* and R^* . (25%)