※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Answer the following questions about a linear programming.

(a) The following tableau is obtained when solving a linear programming with nonnegative variables x_1, x_2, and x_3. The objective function is maximized and slack variables x_4 and x_5 were added.

B.V.	x1	x2	x3	x4	x5	RHS
Z	**a**	0	**b**	0	3	40
x4	-4	0	2	1	6	**c**
x2	-1	1	6	0	-5	12

Provide examples (values of **a**, **b**, and **c**) so that the following statements are true. Also, justify your answers.

i. The current solution is a feasible solution. (5%)

ii. The linear programming is unbounded. (5%)

iii. The current solution is optimal but there are alternative optimal solutions. (5%)

(b) For a sensitivity analysis of a linear programming, briefly describe what are the 100% optimality and feasibility rules? (5%)

2. (15%) National Cheng Kung University is now building a recreational facility. It is known that this project consists of eight major activities (A, B, C,..., H). Suppose we may estimate activity durations accurately. The duration of each activity and the precedence relations are given in the following table:

Activity	A	B	C	D	E	F	G	H
Predecessor	-	-	A	B	B	C, D	D, E	C, E
Duration (Weeks)	16	28	8	12	24	12	36	32

Use the information from the table to answer the following questions:

(a) Draw a project network and use it to find the critical path of this project. (5%)

(b) The project manager needs to reduce the project completion time to **80** weeks. The manager can expedite these activities so that the project can be finished on schedule. The extra cost to expedite each activity (per week) and the maximum reducible time is given in the following table.

Activity	A	B	C	D	E	F	G	H
Extra Cost (thousands)	20	25	30	30	10	10	6	10
Maximum reducible weeks	3	3	4	5	8	3	10	10

Develop a linear programming model to help him make the decision including the start time and the amount time by which an activity time is crashed with minimum extra cost. Clear define your decision variables, objective function, and constraints (**DO NOT** solve the LP). (10%)

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3. (15%) Consider the following non-linear programming model:

Minimize $f(\mathbf{x}) = (x_1 - 4)^2 + (x_2 - 8)^2$

subject to: $g_1(\mathbf{x}) = 2x_1^2 + 2x_2^2 \le 10$

$g_2(\mathbf{x}) = x_1 \le 3$

$g_3(\mathbf{x}) = x_1 \ge 0$

$g_4(\mathbf{x}) = x_2 \ge 0$

(a) Verify that if this is a convex programming problem. (5%)

(b) What are the KKT conditions for this problem? (5%)

(c) Solve the problem by the conditions you found in (b). (5%)

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4 (25%) NCKU Airlines is a small airline that operates flights from Taipei to Tainan. We will analyze the behavior of its airport ticket counter. The interarrival times of customers to the ticket counter follow an exponential distribution with a mean of 4 minutes. There is one ticket agent at the counter. 25% of the time his service time is exactly 1 minute, 50% of the time his service time is exactly 3 minutes, and 25% of the time his service time is exactly 5 minutes.

[TIP: This problem will go most smoothly if you keep everything in minutes]

a) (6 points) What is the probability that exactly 3 customers arrive to the ticket counter in 8 minutes (Show the function; no numerical answer required)?

b) (12 points) We are going to simulate from this system:

(i) (3 points) Using the following uniform random numbers in order -- 0.7, 0.2, 0.9 -- simulate the first three **interarrival** times.

If it helps (because you don't have calculator) you may use the following table to calculate ln(x) or e^x if you need them:

x	ln(x)	e^x
0.1	-2.30	1.11
0.2	-1.61	1.22
0.3	-1.20	1.35
0.4	-0.92	1.49
0.5	-0.69	1.65
0.6	-0.51	1.82
0.7	-0.36	2.01
0.8	-0.22	2.23
0.9	-0.11	2.46

(ii) (3 points) Using the following uniform random numbers in order -- 0.8, 0.5, 0.1 -- simulate the first three **service** times.

(iii) (6 points) Assuming that the system begins empty (no customers at the ticket counter), perform a simulation that determines at what time the third customer completes service. For full credit, please show all of your work.

c) (7 points) Now using formulas (not your simulation results), to estimate the expected length of the queue at the ticket counter in the long run.

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5 (25%) An industrial engineering intern at NCKU, Inc. is not allowed to have more than 3 work assignments on his to-do list at any one time. He works on the assignments one at a time in the order he receives them. The rate that his boss comes up with new assignments follows a Poisson process with a mean of 1 per day. If the boss comes up with a new assignment but the intern already has 3 assignments on his to-do list, he gives the assignment to someone else.

The rate that the intern completes assignments depends on how many assignments he has on his to-do list (i.e. he stretches out the work to fill the time). If he has three assignments on his to-do list, the time it takes him to complete an assignment has a mean of 1 day; if he has two assignments, his completion time has a mean of 2 days; and if he has one assignment, his completion time has a mean of 3 days. All of these times are exponential.

a) (10 points) Calculate the steady state probabilities π.
b) (15 points) Calculate λ_{eff} (effective arrival rate), L, L_q, W, W_q.