

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

I. Formulation and analyses

1. How do you detect infeasibility, unboundedness, and multiple solutions in the simplex method? Give your reasons. (20%)
2. A company has m plants to manufacture a type of product for its n stores to sell. The i th plant, $i=1, \dots, m$, can manufacture an amount of at most a_i each month, and the j th store, $j=1, \dots, n$, needs an amount of at least b_j each month. The unit shipping cost from plant i to store j is c_{ij} . Formulate the problem of shipping the product manufactured by the m plants to the n stores (as a linear program) that will minimize the total shipping cost. Also formulate the dual of this linear program. (15%)
3. You have a budget of B to buy n commodities. Your utility of consuming x_i quantity of commodity i is $U(x_1, x_2, \dots, x_n)$, which is assumed to be a concave function. Suppose the unit price of commodity i is p_i . How would you allocate the budget to the n commodities so that your utility will be maximized? You need to formulate the problem first; then derive the solution. (15%)

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II.1 (15%) Suppose X has a binomial distribution with parameters p and N , where N has a Poisson distribution with mean λ , derive the marginal distribution for X .

II.2 (15%) You are given the following payoff table (in units of thousands of dollars) for the production planning of two types of rice in a decision analysis problem.

Rice type	State of Nature (Rain fall in the year)		
	Above average S_1	Average S_2	Below Average S_3
R_1	220 tons)	170	110
R_2	200	180	150
Prior probability	0.6	0.3	0.1

- Which alternative should be chosen under the Maximin payoff criterion?
- Which alternative should be chosen under the Bayes' decision rule?
- Using Bayes' decision rule, if the true probability of the state of nature S_3 is 0.1, perform sensitivity analysis with respect to the prior probabilities of states S_1 and S_2 to determine the crossover point where the decision shifts from one alternative to the other.

II.3 (20%) A production process changes states in accordance with the following transitional probability matrix in a Markov chain:

$$P = \begin{array}{c|cccc} 1 & 0.9 & 0.1 & 0 & 0 \\ 2 & 0 & 0.8 & 0.1 & 0.1 \\ 3 & 0 & 0 & 0.7 & 0.3 \\ 4 & 0.9 & 0 & 0 & 0.1 \end{array}$$

Where States 1, 2, 3 denote the acceptable states (up) and state 4 denotes the unacceptable one (down).

Determine

- The rate of breakdown (the rate at which the process goes from up to down)
- The expected length of time the process remains up when it goes up.
- The expected length of time the process remains down when it goes down.