

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let  $X$  be a random variable with probability density

$$f(x) = \begin{cases} c(1-x^2), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of  $c$ ? (5%)

(b) What is the cumulative distribution function of  $X$ ? (5%)

2. Let  $X$  be uniform over  $(0, 1)$ . Find  $E[X|X < \frac{1}{4}]$ . (5%)

3. Suppose that the joint probability density of  $X$  and  $Y$  is

$$f_{X,Y}(x, y) = \lambda^2 e^{-\lambda y}, \quad 0 < x < y < \infty$$

(a) Find the density of  $X$ . (5%)

(b) Find the density of  $Y$ . (5%)

4. Suppose 38% of doctors think their patients receive unnecessary medical care and a sample of 300 doctors were taken.

(a) Show the sampling distribution of proportion of the doctors who think their patients receive unnecessary medical care. (5%)

(b) What is the probability that the sample proportion will be within  $\pm 0.03$  of the population proportion? (5%)

5. Given the following sampling data:

98	100	105	108	109	110	110	111	112	113
115	115	118	118	119	120	121	123	123	125
128	135	136	137	138	144	145	146	148	150

(a) Develop a 95% confidence interval estimate of the population mean. (10%)

(b) At 95% confidence interval, how large a sample would provide a margin of error of 5? (5%)

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6. A teacher teaches statistics in two different classes using two different teaching methods. He is wondering if using two different teaching methods affects students' final exam scores? In the following table, sample 1 shows the exam scores from 8 students in the first class and sample 2 shows 10 students' scores from the second class.

Sample 1	55	66	67	58	69	71	72	64		
Sample 2	66	68	69	69	70	71	71	80	82	66

- (10%) Formulate the hypotheses that can be used to determine if the sample data support the hypothesis that the average final exam scores from students in these two classes are different? Use  $\alpha = 0.05$ . Explicitly present your null hypothesis, test procedure and conclusion.
7. An engineer is investigating the effect of three different ingredients (ingredient 1, 2, and 3) on the reaction time of a chemical process. Three operators are chosen for this experiment. A randomized complete block design (RCBD) is conducted and the result is given in the following table.

ingredient	Operator (Block)		
	1	2	3
1	10.5	11.2	10.8
2	12.1	14.8	12.2
3	13.6	13.8	13.9

- (1) (5%) Explain why it is better to use a RCBD in this case?
- (2) (10%) Complete a ANOVA table to test for differences between ingredients used. Use  $\alpha = 0.05$ . Explicitly present your null hypothesis and conclusions.
- (3) (2%) What is the value of residual from the response with **ingredient 1** and **operator 2**?
- (4) (3%) In your analysis, what assumptions regarding the distribution of residuals need to be verified?

8. In a chemical process, it is known that two important factors including temperature ( $x_1$ , °C) and using catalyst in the process or not ( $x_2 = 0$  if no and  $x_2 = 1$  if yes) would influence if the yield of the process. If the yield of a process can exceed the required standard, the engineer records the response variable  $Y = 1$  and  $Y = 0$  otherwise. Answer the following questions.

- (1) (5%) Explain why a typical multiple regression model  $Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$  is not appropriate to predict the response variable in this case?
- (2) (10%) Suppose there are  $n$  outputs  $Y_i$ ,  $i = 1, 2, \dots, n$  under test conditions  $(x_{1i}, x_{2i})$ ,  $i = 1, 2, \dots, n$  available. A multiple logistic regression model with logit link function and two predictors in first-order is assumed to be appropriate. Write down the form of log-likelihood function that is used to find the maximum likelihood estimates for parameters.
- (3) (5%) The original data is not given here but a report table generated from computer software is given:

Logistic Regression Table:

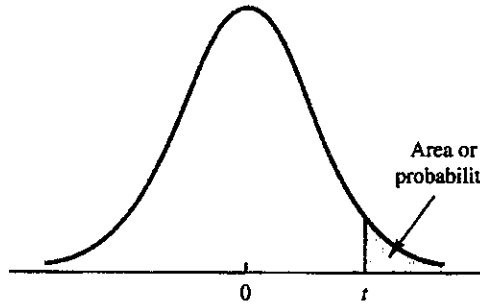
Coefficients

Term	Coef	SE Coef	VIF
Constant	-10.00	2.17	
x1 (Temperature)	0.1326	0.0324	1.05
x2 (with catalyst or not)	1.764	0.539	1.05

It is known that when the temperature is 75°C and there is no catalyst used in the process, the probability of the yield from the chemical process can exceed the required standard is about 48.5%. Based on the table, predict the probability of the yield from the chemical process can meet the standard when the temperature is still 75°C but some catalyst is used in the process.



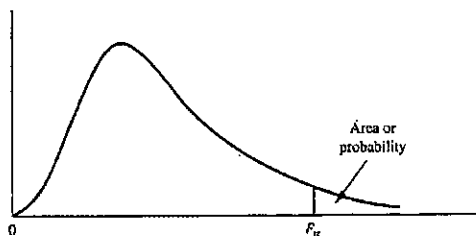
t-distribution table



Entries in the table give  $t$  values for an area or probability in the upper tail of the  $t$  distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail,  $t_{.05} = 1.812$ .

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756

F-distribution table



Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom											
		1	2	3	4	5	6	7	8	9	10	15	20
1	.10	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	61.22	61.74
	.05	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	245.95	248.02
	.025	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28	968.63	984.87	993.08
	.01	4052.18	4999.34	5403.53	5624.26	5763.96	5858.95	5928.33	5980.95	6022.40	6055.93	6156.97	6208.66
2	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.42	9.44
	.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.43	39.45
	.01	98.50	99.00	99.16	99.25	99.30	99.33	99.36	99.38	99.39	99.40	99.43	99.45
3	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.20	5.18
	.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.25	14.17
	.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	26.87	26.69
4	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.87	3.84
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.66	8.56
	.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02
5	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.24	3.21
	.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.43	6.33
	.01	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.72	9.55
6	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.87	2.84
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.27	5.17
	.01	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.56	7.40
7	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.63	2.59
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.57	4.47
	.01	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.31	6.16
8	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.46	2.42
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.10	4.00
	.01	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.52	5.36
9	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.34	2.30
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.77	3.67
	.01	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.96	4.81
10	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.24	2.20
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77
	.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.52	3.42
	.01	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.56	4.41
11	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.17	2.12
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.33	3.23
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.25	4.10
12	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.10	2.06
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.18	3.07
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.01	3.86