國立成功大學 107 學年度碩士班招生考試試題

系　　所：工業與資訊管理學系

考試科目：作業研究　　考試日期：0206，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%)

(i) Is it true that if a function of n variables $f(x_1, x_2 \ldots, x_n)$ is convex, then the set $\{\mathbf{x} | f(x_1, x_2 \ldots, x_n) \le c\}$ where c is a constant is a convex set? If yes, give a proof. If no, give a counterexample. (5%)

(ii) Consider the following non-linear programming model:

Minimize $f(x_1, x_2) = 2x_1^2 + 2x_2^2 + bx_1x_2$

subject to: $g_1(x_1, x_2) = 3x_1^2 + 3x_2^2 \le 24$

$g_2(x_1, x_2) = x_1 \le 2$

(a) Give the range of value b in $f(x_1, x_2)$ so that this is a convex programming problem. (5%)

(b) Let $b = -6$ in $f(x_1, x_2)$. Write down the KKT conditions for this problem? (10%)

(c) Find the local minimum points by the conditions you found in (b). (10%)

2. (20%) A senior student at the industrial and information management department has 5 days remaining before his final examinations begin in his three courses. Since every course is equally important, he needs at least one day studying for each course and he likes to concentrate on just one course each day. He wants to make a study plan to allocate his time effectively. Therefore, he would like to allocate 0,1, or 2 extra days to each course. He estimates the number of grade points for all possible allocations for each course, as shown in the following table:

	Estimated grade points		
Extra study days	Course1	Course 2	Course 3
0	2	2	2
1	2	3	2
2	3	4	3

The senior student recalls the dynamic programming method learned in the Operation Research course and decide to use this method to find the optimal study plan.

He defines stage n in this case corresponds to course n, $n = 1, 2, 3$. The initial state S_n is the number of extra days still available for allocation to the remaining courses (i.e., $S_1 = 2$). The decision variable x_n, $n = 1, 2, 3$ at each stage are the number of extra study days allocated to course n. The goal is to maximize the total estimated grade points to be obtained in his final exam. Find his optimal study plan by dynamic programming using the notations defined above.

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3. (25 points) You have been hired as a consultant by NCKU, Inc. Their sales fluctuate between two levels – high and low. The sales level is heavily dependent on whether or not the company advertises. If the company advertises in a given quarter the probability that the following quarter's sales are high is 1/2 or 3/4 depending on whether the current quarter's sales are low or high respectively. If the company does not advertise in a given quarter, then those probabilities will drop to 1/4 and 1/2.
If the quarterly sales are low, the profit (before advertising costs) is $2 million. If the quarterly sales are high, the profit (before advertising costs) is $4 million. Advertising (when done) costs $1 million per quarter.
They would like to analyze the following 3 advertising strategies:

- Strategy 1: Always advertise
- Strategy 2: Never advertise
- Strategy 3: Advertise when sales are low, but not when sales are high

a) (3 points) If we model this as a Markov Chain,
   - What is the stochastic process X_n?
   - What does the time index n represent?
   - What is the state space and what do the states represent?

b) (6 points) What is the transition matrix P for each of the three strategies?

c) (15 points) Your client would like to see the following chart completed for the three strategies. Please show all calculations.

	Strategy 1	Strategy 2	Strategy 3
Long-run probability of a low sales quarter			
Long-run probability of a high sales quarter			
Expected quarterly profit (before advertising costs)			
Expected quarterly advertising costs			
Expected quarterly profit (after advertising costs)			

d) (1 points) Based on your results, which strategy would you recommend?

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4. (25 points) Customers arrive to a hotel taxi stand according to a Poisson process with rate 10 per hour. All taxis go to the airport and because of a local ordinance, the taxi stand can only accommodate four customers. Taxis arrive to the stand on average every 4 minutes (exponentially distributed). Each taxi can hold up to two people (so if there are two or more customers at the taxi stand, the taxi will take two people. If only one customer is at the stand it will just take the one).

a) (5 points) Model this as a Continuous Time Markov Chain. Define your stochastic process Y(t), time index t, state space, and transition rate matrix G
b) (5 points) Find the transition matrix P for the embedded DTMC and the expected holding times in each state. [Numerical Answer]
c) (5 points) How would I calculate the expected number of customers at the stand at the end of 4 hours given the stand starts empty? [No numerical answer required]
d) (5 points) Write out the balance equations to solve for steady-state probability π.
e) (5 points) What is the probability that fewer than two taxis arrive between 4:00pm-4:15pm? [Note: Taxis arrive according to a Poisson Process. No numerical answer required]