

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

I.1. (20%) Consider the following problem and its resulting final simplex tableau.

Maximize $Z = ax_1 + bx_2 + cx_3$	B.V.	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	RHS
subject to	$Z$	0	2	0	$\frac{1}{5}$	$\frac{3}{5}$	$d$
$6x_1 + 3x_2 + 5x_3 \leq i$	$x_1$	1	$e$	0	$g$	$-\frac{1}{3}$	$\frac{5}{3}$
$3x_1 + 4x_2 + 5x_3 \leq j$	$x_3$	0	$f$	1	$h$	$\frac{2}{5}$	3
$x_1, x_2, x_3 \geq 0.$							

Identify the value of  $a, b, c, d, e, f, g, h, i$  and  $j$ .

I.2. (20%) NCKU company makes three products. Each production run of product  $i$  involves a fixed cost  $F_i$  and a per-unit cost  $c_i$ . The unit revenue for product  $i$  is  $r_i$ . These products need two production processes. The time requirement and availabilities for each process are given as follows:

Process	Product			Hours available
	1	2	3	
I	0.25	0.2	0.3	300
II	0.4	0.5	0.2	400

NCKU will upgrade exactly one of two processes. The upgrade will raise the number of available hours by 20% for Process I and 10% for Process II. Formulate a mathematical Programming model to determine which process to upgrade and the production levels to maximize the profit.

I.3. (10%) Consider the following problem:

$$\text{Maximize } Z = 5x_1 - x_1^2 + 8x_2 - x_2^2 + 10x_3 - x_3^2 + 15x_4 - x_4^2 + 20x_5 - x_5^2,$$

subject to

$$x_1 + x_3 + x_4 \leq 25,$$

$$x_1 \in \{3, 6, 12\}, x_2 \in \{3, 6\}, x_3 \in \{6, 12\}, x_4 \in \{3, 6, 9, 12\}, x_5 \in \{9, 12, 15, 16\},$$

and all these variables must have different values.

Use the techniques of constraint programming (domain reduction, constraint propagation, a search procedure, and enumeration) to identify all the feasible solutions and then to find an optimal solution.

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II.1 (15%) Data indicates that the number of traffic accidents in NCKU on a rainy day is a Poisson random variable with mean 9, whereas on a dry day it is a Poisson random variable with mean 3. Let  $X$  denote the number of traffic accidents tomorrow. The central weather bureau forecasts that it will rain tomorrow with probability 0.6, find

- The expected value  $E(X)$ ;
- The probability  $P\{X=0\}$ ;
- The variance  $\text{Var}(X)$

II.2 (15%) Wafers arrive at an IC fabrication facility and wait in the buffer area until a total number of  $k$  wafers have accumulated. Upon the arrival of the  $k^{\text{th}}$  wafer, all wafers are instantaneously processed by the machine, and the process repeats. Let  $x_k$ ,  $k = 1, 2, \dots$ , denote the arrivals of wafer in successive periods, assumed to be independent random variable whose distribution is given by  $\Pr\{x_k = 0\} = \alpha$ , and  $\Pr\{x_k = 1\} = 1 - \alpha$ , where  $0 < \alpha < 1$ . Let  $X_n$  denote the number of wafers in the system at time  $n$ .

- State assumptions required so that the above production problem can be modeled as a Markov chain;
- Define state space and show that  $\{X_n; n = 0, 1, 2, \dots\}$  is a Markov Chain;
- Derive the transition probabilities and the transition probability matrix.

II.3 (20%) Customers arrive at a service station according to a Poisson process of rate  $\lambda$  customer/hour. Let  $X(t)$  be the number of customers that have arrived up to time  $t$ . Consider a fixed time  $s$  for  $0 < s < t$ , determine

- The conditional probability  $P\{X(t) = n+k \mid X(s) = n\}$ ;
- The expected value  $E[X(t)X(s)]$ .