國立成功大學 109 學年度碩士班招生考試試題

系　所：工業與資訊管理學系

考試科目：作業研究

考試日期：0211，節次：2

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

I.1. (20%) Consider the following linear programming problem and its resulting final simplex tableau.

Maximize $Z = ax_1 + bx_2$

subject to $x_1 \le c$

$x_2 \le 6$

$6x_1 + dx_2 \le 36$

$x_1, x_2 \ge 0$.

Given that $0 < a < b$, $0 < d < 6$, and $c + d > 6$, its resulting final simplex tableau is given as:

B.V.	x_1	x_2	x_3	x_4	x_5	RHS
Z	0	0	0	10/3	2/3	e
x_3	0	0	1	2/3	−1/6	3
x_2	0	1	0	1	0	6
x_1	1	0	0	−2/3	1/6	2

Identify the values of a, b, c, d, and e.

I.2. (15%) Given 4 decision alternatives and 4 states of nature, consider the following **Profits** in the payoff table.

	State 1	State 2	State 3	State 4
Alternative1	-50	50	90	125
Alternative2	-20	35	80	160
Alternative3	-40	70	100	110
Alternative4	-60	20	85	140

(i) Which decision alternative should be chosen using the Maximin payoff criterion?

(ii) Which decision alternative should be chosen using the Minimax Regret criterion?

Consider the following prior probabilities for the states of nature.

	State 1	State 2	State 3	State 4
Probability	0.1	0.5	0.2	0.2

(iii) Which decision alternative should be chosen using the Bayes' desicion rule?

(iv) What is the highest amount you would be willing to pay for the perfect information (assuming decisions are based on the Bayes' desicion rule)?

(v) Suppose the true probabilities for the states of nature 3 and 4 (State 3 & State 4) are both 0.2, let p be the prior probability of States 1. At which value of p does the optimal decision alternative shifts from one to another using Bayes' desicion rule?

(a) $p = 0.25$　(b) $p = 0.38$　(c) $p = 0.53$　(d) $p = 0.59$.

I3. (15%) Consider the following non-linear programming model:

$$\text{Minimize } f(\mathbf{x}) = x_1^2 + x_2^2 - 4x_1 - 4x_2 + 3$$

$$\text{subject to: } g_1(\mathbf{x}) = 2x_1^2 + 2x_2^2 - 12 \le 0$$

Write down the Lagrange function and the KKT conditions for this problem and find the points satisfying these conditions.

II

1. People arrive at a barber shop according to a Poisson process with a mean of one person per hour. The time taken by the barber for a haircut is an exponential random variable with a mean of thirty minutes. (a) Suppose the barber does not allow people to enter when he is busy, calculate L (the expected number of customers in the barber shop), W (the expected time that a customer spends in the barber shop), and P_0 (the probability that no customer is in the barber shop) (b) Suppose the barber works for ten hours a day. How many customers are lost every day if he does not allow people to enter when he is busy? (15%)

2. Consider a deterministic inventory problem which does not allow stock out. Denote d as the demand per unit time, k as the setup cost for ordering one batch of amount Q, c as the unit cost of the commodity, h as the inventory holding cost per unit per unit time. (a) Formulate the total cost TC per unit time if an amount Q is ordered each period, and find the optimal Q^*, i.e. the economic order quantity (EOQ). (b) If the cost of the commodity cd is ignored ($\overline{TC} = TC - cd$), show that the optimal total cost is $\overline{TC}^* = \sqrt{2hkd}$, and the ratio of the total cost associated with a non-optimal quantity Q to that associated with the optimal quantity Q^* is $\overline{TC}/\overline{TC}^* = (Q^*/Q + Q/Q^*)/2$. (c) If the order quantity Q is twice of the optimal order quantity Q^*, then how much percent of the total cost will be increased as compared to the optimal cost $\overline{TC}^*$? (20%)

3. For a Markov chain with the one-step transition probability matrix $\boldsymbol{P}$ shown on the right, (a) find $\boldsymbol{P}^* = \lim_{n\to\infty} \boldsymbol{P}^{(n)}$. (b) Starting with state 1, what is the probability of being in state 1 in the long run? (c) Suppose the initial state of being in state [0, 1, 2, 3, 4] is selected randomly according to the probability of [0.1, 0.2, 0.3, 0.4, 0]. What is the probability of being in state 1 in the long run? (15%)

$$\boldsymbol{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 1/4 & 3/4 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$