國立成功大學

110學年度碩士班招生考試試題

編　號：242

系　所：工業與資訊管理學系

科　目：作業研究

日　期：0203

節　次：第 2 節

備　註：可使用計算機

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I.1 (10%) For the following linear programming problem [LP]: $Z_{LP} = \text{Minimize}\{CX : AX \geq b, X \in R_+^n\}$

- State the assumptions made in formulating [LP].
- State the weak duality theorem.
- State the complementary slackness conditions.

I.2 (20%) For the following Linear Programming problem [LP]: $Z_{LP} = \text{Maximize}\{CX : AX \leq b, X \in R_+^n\}$, using duality theorems and Primal-Dual relationship, show that: Either $\{X \in R_+^n : AX \leq b\} \neq \varnothing$, or (exclusively) there exists: $\{V \in R_+^m$ such that $VA \geq 0$, and $Vb < 0\}$.

I.3 (20%) For the following non-linear programming problem: Maximize $f(x) = 12x - 3x^4 - 2x^6, 0 \leq x \leq 2$, apply Bisection method or Newton's method (choose only one) for 5 iterations to find the near optimal solution. The initial value for Newton's method is x_0 =1.0. Show your intermediate computations and objective values with a table.

II.1 (25%) A machine shop has two identical machines, which are supervised by one operator. Given that a machine is operational at the start of a day, it has a probability of 0.4 of breaking down sometime during the day. At the start of each day, if there is a machine that is broken down, the operator will spend the entire day repairing it, and the machine will be operational at the end of the day. However, the operator can only repair one machine in a single day. The two machines are independent of each other.

a) (10 points) Model this problem as a Markov chain. Draw the states and transitions, and include the probabilities on each arc. Each state should represent the number of machines that are broken down at the end of a day.
b) (10 points) Compute the steady state probabilities.
c) (5 points) The machine shop's profit per day is determined by the number of machines that are operational at the end of the day. The profit is $250 if both machines are operational, $200 if one machine is operational, and $100 if no machine is operational. Determine the machine shop's expected profit on a randomly chosen day far into the future.

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II.2 (25%) At a particular emergency department, patients arrive complaining of migraines. There is one doctor who sees these patients one after another, and the time she takes to see one patient is exponentially distributed with a mean of 6 minutes. The doctor will never run out of patients to see. For each patient, independent of other patients, there are two equally likely outcomes after they have finished being seen by the doctor: either the doctor will order a CT scan, or the doctor will require the patient to wait for some length of time before being allowed to leave. For patients who require a CT scan, there are two CT machines available, and the time to complete a CT scan is exponentially distributed with mean 20 minutes. For patients who are required to wait before being allowed to leave, the length of time that a patient has to wait is exponentially distributed with a mean of 15 minutes; the patient will leave the emergency department as soon as this length of time has passed. Assume that the doctor and the CT machines are all working 24 hours a day, 7 days a week.

a) (7 points) For patients requiring a CT scan, describe the queuing system that can be used to model the wait for the two CT machines. For patients who are required to wait before being allowed to leave, describe the queuing system that can be used to model the wait before leaving. In both cases, clearly identify the interarrival and service distributions, their rates, the number of servers, and any other relevant information.

b) (4 points) Determine the expected number of patients in the emergency department who have been seen by the doctor and have been required to wait before leaving.

c) (6 points) Determine the probability that there is at least one free CT machine.

d) (8 points) Determine the expected length of time that a patient will spend waiting after being seen by the doctor (in this question, "waiting" means either waiting for a CT machine, or waiting until being allowed to leave the emergency department.)