國立成功大學
112學年度碩士班招生考試試題

編　號： 235

系　所： 工業與資訊管理學系

科　目： 作業研究

日　期： 0207

節　次： 第 2 節

備　註： 可使用計算機

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. [15%] The Cost-Less Corp. supplies its four retail outlets from its four plants. The shipping cost per shipment from each plant to each retail outlet is given below.

		Unit Shipping Cost Retail Outlet			
		1	2	3	4
Plant	1	\$500	\$600	\$400	\$200
	2	\$200	\$900	\$100	\$300
	3	\$300	\$400	\$200	\$100
	3	\$200	\$100	\$300	\$200

Plants 1, 2, 3, and 4 make 10, 20, 20, and 10 shipments per month, respectively. Retail outlets 1, 2, 3, and 4 need to receive 20, 10, 10, and 20 shipments per month, respectively. The distribution manager, Randy Smith, now wants to determine the best plan for how many shipments to send from each plant to the respective retail outlets each month. Randy's objective is to minimize the total shipping cost.

Starting with the initial basic solution from the northwest corner rule, interactively apply the transportation simplex method to obtain an optimal solution and the optimal value for the distribution manager.

2. Consider the problem

$$
\begin{aligned}
\min \quad & x_1 + 2x_2 \\
\text{s.t.} \quad & x_1 + 2x_2 = 3 \\
& x_1 \leq -1 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

(a) [10%] Write down and solve its dual problem.

(b) [5%] Use the optimality of its dual problem to describe that the original problem is infeasible.

3. [20%] Using simplex method to check the optimality of the following linear programming

$$
\begin{aligned}
\max \quad & 5x_1 + x_2 + 3x_3 + 4x_4 \\
s.t. \quad & x_1 - 2x_2 + 4x_3 + 3x_4 \leq 20 \\
& -4x_1 + 6x_2 + 5x_3 - 4x_4 \leq 40 \\
& 2x_1 - 3x_2 + 3x_3 + 8x_4 \leq 50 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

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4. (25 points) We model the movement of the taxi as a Markov chain $\{X_n, n = 0,1,\ldots\ldots\}$, where the time index n represents the number of riders the taxi has transported (groups of riders count as one), and X_n represents the region of the city that is the destination of the nth rider. When the taxi delivers a rider, it stays in the destination region until it picks up another rider. The city has three regions, so the state space is $M = \{1,2,3\}$. Based on data collected on the movement of several taxis, the following one-step transition matrix has been estimated:

$$\mathbf{P} = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.5 & 0.2 & 0.3 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}.$$

The other matrixes are provided as follows:

$$\mathbf{P}^2 = \begin{bmatrix} 0.48 & 0.27 & 0.25 \\ 0.43 & 0.28 & 0.29 \\ 0.39 & 0.27 & 0.34 \end{bmatrix}, \qquad \mathbf{P}^3 = \begin{bmatrix} 0.429 & 0.273 & 0.298 \\ 0.443 & 0.272 & 0.285 \\ 0.456 & 0.273 & 0.271 \end{bmatrix}$$

$$\mathbf{P}^4 = \begin{bmatrix} 0.444 & 0.273 & 0.283 \\ 0.44 & 0.273 & 0.287 \\ 0.436 & 0.273 & 0.291 \end{bmatrix}, \qquad \mathbf{P}^5 = \begin{bmatrix} 0.439 & 0.273 & 0.288 \\ 0.441 & 0.273 & 0.286 \\ 0.442 & 0.273 & 0.285 \end{bmatrix}$$

At the beginning of the day, 40% of the taxis are assigned to start in region 1, 30% in region 2, and the remainder in region 3. Please calculate the following quantities:

a) (5 points) For a taxi that its second rider takes it to region 2, what is the probability that its fourth rider takes it to region 3? (i.e., $\mathrm{P}(X_4 = 3 \mid X_2 = 2)$)

b) (5 points) For a taxi that starts in region 1 and the second rider takes it to region 3, what is the probability that its fourth rider takes it to region 1? (i.e., $\mathrm{P}(X_4 = 1 \mid X_0 = 1, X_2 = 3)$)

c) (5 points) For *any* taxi, what is the probability that its fourth rider takes it to region 3?

d) (5 points) The probability that the taxi is in region 3 after the third ride and in region 1 after the fourth ride given that it starts in region 2. (i.e., $\mathrm{P}\left(X_3 = 3, X_4 = 1 \mid X_0 = 2\right)$)

e) (5 points) Given the taxi starts in state 1, the probability of the next five rides going (in order) to regions 2, 3, 1, 3, 2.

5. (25 points) A retail store manages the inventory of washing machines using a (q, Q)-policy, which works as follows: When the number of machines in stock decreases to a fixed value q, an order is placed with the manufacturer for Q new washing machines. It takes a random amount of time for the order to be delivered; the mean delivery time is 2 days (also exponential). If the inventory is at most q when an order is delivered (including the newly delivered order), another order for Q items is placed immediately. Demands for washing machines occur according at a rate of 2 per day (time between demands is exponential). Demands that are not immediately satisfied are lost. Assuming that q = 2 and Q = 1, answer the following questions:
CTMC (Continuous Time Markov Chain)

(a) (5 points) Construct an appropriate CTMC to describe this system. Assume that all inter-event times are independent and exponentially distributed. That is, describe the state variable, state space, and the transition rate diagram.

(b) (10 points) If there are currently two washing machines in the store, what is the probability that the store runs out of washing machines before it has the maximum number of washing machines in stock?

(c) (10 points) Compute the long-run average inventory in the store.