國立成功大學
114學年度碩士班招生考試試題

編　號：168

系　所：工業與資訊管理學系

科　目：作業研究

日　期：0211

節　次：第 2 節

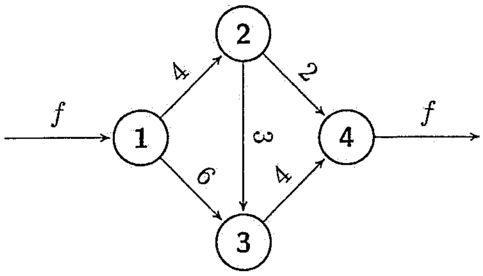
注　意：1.可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. Consider the following linear programming:

$$(P)\quad \begin{array}{ll} \min & 4x_5 + 6x_6 + 3x_7 + 2x_8 + 4x_9 \\ \text{s.t.} & -x_1 + x_2 + x_5 - x_{10} = 0 \\ & -x_1 + x_3 + x_6 - x_{11} = 0 \\ & x_2 + x_3 + x_7 - x_{12} = 0 \\ & -x_2 + x_4 + x_8 - x_{13} = 0 \\ & -x_3 + x_4 + x_9 - x_{14} = 0 \\ & x_1 - x_4 = 1 \\ & x_i \geq 0, i = 1, 2, \cdots, 14 \end{array}$$

(a) [10%] Formulate the dual problem corresponding to (P).

(b) [5%] For the network shown below, apply the primal-dual method as outlined in the slides to determine the flow pattern that achieves the maximum flow from the source(node 1) to the sink(node 4). The arc capacity from node i to node j is represented by the number at the center of the arc connecting these nodes.



Now the initial flow is given by

Node i	Node j	Flow from Node i to Node j
1	2	4
1	3	1
2	3	3
2	4	1
3	4	3

Determine the maximum flow, starting from the given initial flow.

(c) [10%] Find the optimal value for (P). (Hint: Consider the relation between the problems in (a) and (b).)

(d) [5%] Find an optimal solution for (P).

2. Consider the following linear programming:

$$(P)\quad \begin{array}{ll} \max & (-2 + 2\theta)x_1 + (-3 + \theta)x_2 + (-4 - \theta)x_3 \\ s.t. & -x_1 - 2x_2 - x_3 + x_4 = -3 \\ & -2x_1 + x_2 - 3x_3 + x_5 = -4 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0. \end{array}$$

where θ can be assigned any positive or negative values. After apply the simplex method with $\theta = 0$, the final tableau is

Basic Vars	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	1	0	0	$\frac{9}{5}$	$\frac{8}{5}$	$\frac{1}{5}$	$-\frac{28}{5}$
x_1	0	1	0	$\frac{7}{5}$	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{11}{5}$
x_2	0	0	1	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$

(a) [10%] Determine α_1 and α_2 such that over the range $\theta \in [\alpha_1, \alpha_2]$ the above basic solution will remain optimal. Find the objective value $f(\theta)$ (as a function of θ), and then find the best choice θ^* within this range.

$\alpha_1 =$ ________, $\alpha_2 =$ ________, $f(\theta) =$ ________, $\theta^* =$ ________

(b) [5%] Given that θ within the range of values found in (a), identify the shadow prices $S_1(\theta)$ and $S_2(\theta)$ (as a function of θ) for the two resources.

$S_1(\theta) =$ ________, $S_2(\theta) =$ ________

(c) [5%] Given $\theta = 0$, additional constraint $x_1 + x_2 + x_3 \geq 4$ is added to the problem (P). Determine the optimal solution $(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*)$ of (P)

$(x_1^*, x_2^*, x_3^*, x_4^*, x_5^*) =$ ________

3. Please express the following statements with linear constraints using binary variables.

(a) (5%) $x_1 + x_2 \leq 2$ or $2x_1 + 3x_2 \geq 8$.

(b) (5%) x_3 can be values 0, 5, 9, or 12.

(c) (5%) If $x_4 \leq 4$, then $x_5 \geq 6$; otherwise $x_5 \leq 3$.

4. There are two types of customers. Type 1 and 2 customers arrive in accordance with independent Poisson processes with respective rate λ_1 and λ_2. There are two servers. A type 1 arrival will enter service with server 1 if that server is free; if server 1 is busy and server 2 is free, then the type 1 arrival will enter service with server 2. If both servers are busy, then the type 1 arrival will go away. A type 2 customer can only be served by server 2; if server 2 is free when a type 2 customer arrives, then the customer enters service with that server. If server 2 is busy when a type 2 arrives, then that customer goes away. Once a customer is served by either server, he departs the system. Service times at server i are exponential with rate μ_i, $i = 1, 2$. Suppose we want to find the average number of customers in the system.

   (a) (5%) Define states.

   (b) (5%) Give the balance equations. Do not attempt to solve them.

   In terms of the long-run probabilities, what is

   (c) (5%) the average number of customers in the system?

   (d) (5%) the average time a customer spends in the system.

5. (15%) Consider a population of individuals each of whom possesses two genes which can be either type *A* or type *a*. Suppose that in outward appearance type *A* is dominant and type *a* is recessive. (That is, an individual will have only the outward characteristics of the recessive gene if its pair is *aa*.) Suppose that the population has stabilized, and the percentages of individuals having respective gene pairs *AA*, *aa*, and *Aa* are p, q, and r. Call an individual dominant or recessive depending on the outward characteristics it exhibits. Let S_{11} denote the probability that an offspring of two dominant parents will be recessive; and let S_{10} denote the probability that the offspring of one dominant and one recessive parent will be recessive. Compute S_{11} and S_{10} to show that $S_{11} = {S_{10}}^2$.