國立成功大學

114學年度碩士班招生考試試題

編　號：170

系　所：工業與資訊管理學系

科　目：統計學

日　期：0211

節　次：第 3 節

注　意：1.可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

Part I 單選題(50%)

I-1.( ) Denote that $\Pr(\cdot)$ is the probability of an event occurring. If events A and B are two independent events, $\Pr(A)\neq 0$ and $\Pr(B)\neq 0$, which of the following statements is **NOT** correct?

(A) Events A and B are also exclusive.

(B) $\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A)\times\Pr(B)$.

(C) $\Pr(A\mid B)=\Pr(A)$.

(D) $\Pr(B\mid A)=\Pr(B)$.

(E) $\Pr(A\cap B)\neq 0$

I-2.( ) The products in a factory are produced by two machines M_1 and M_2 with known proportions $\Pr(M_1)=70\%$ and $\Pr(M_2)=30\%$. The products produced by these two machines are sometimes flawed and the probabilities of two machines produce nonconforming products are $\Pr(\text{produce nonconforming product}\mid M_1)=15\%$, $\Pr(\text{produce nonconforming product}\mid M_2)=5\%$. If a product is found flawed, what is the probability this product was produced by machine M_1?

(A) 10.5% (B) 87.5% (C) 30% (D) 70% (E) 12.5%.

I-3.( ) A random variable $X\sim Exp(\theta)$. Given that $\Pr(X\geq 15)=0.223$, what is the conditional probability $\Pr(X\geq 30\mid X\geq 15)$?

(A) 0.0223 (B) 0.158 (C) 0.223 (D) 0.777 (E) 0.95.

I-4.( ) A random variable $Y\sim f(y)$ where $f(y)$ is the probability density function. Which of the following distribution has the largest expected value of Y, $\mathrm{E}(Y)$?

(A) $f(y)=1/10,\ y\in[0\quad 10]$.

(B) $f(y)=\frac{1}{6}\times e^{-\frac{y}{6}},\ y>0$

(C) $f(y)=\frac{1}{16\times\Gamma(4)}\times y^3\times e^{-\frac{y}{2}},\ y>0$

(D) $f(y)=\frac{1}{\sqrt{50\pi}}\times e^{\frac{(y-9)^2}{50}},\ y\in\mathrm{R}$

(E) $f(y)=\frac{1}{\sqrt{100\pi}}\times e^{\frac{(y-7.5)^2}{100}},\ y\in\mathrm{R}$

I-5.( ) Which of the following distribution has the largest variance of the random variable, $\mathrm{Var}(X)$?

(A) $X\sim Poisson(\lambda=3)$

(B) $X\sim Binomial(n=10,p=0.35)$

(C) $X\sim Geo(p=0.45)$, where $\Pr(X=x)=p(1-p)^{x-1},x=1,2,3,....$

(D) $X\sim Hypergeometric(N=30,r=15,n=10)$, where $\Pr(X=x)=\binom{r}{x}\binom{N-r}{n-x}\Big/\binom{N}{n}$ and N is the number of elements in the population, n is the number of trials, and r is the number of elements in the population labeled success.

(E) $X\sim N(\mu=10,\sigma=1.673)$.

I-6.( ) Assume that there are 40% (true proportion, p) of a firm's orders come from first-time customers. An analyst decides to take a random sample of 400 orders to estimate the proportion of first-time customers. What is the approximate probability that the sample proportion $\bar{p}$ will be between 35% and 50%?

(A) $\Phi(-2.04)$ (B) $\Phi(4.08)-\Phi(-2.04)$ (C) $\Phi(4.08)$ (D) $1-\Phi(-2.04)$ (E) 1.00.

I-7.( ) An electronical device has lifetime following an exponential distribution, $f(x)=\frac{1}{\theta}\times e^{-\frac{x}{\theta}}, x>0$, with mean lifetime 2,000. An engineer takes a random sample of such device with sample size 400 and calculates the sample average, $\bar{x}$. By central limit theorem, one may assume that the approximate sampling distribution of $\bar{x}$ is ?

(A) $\bar{x}\sim Exp(\theta=400)$ (B) $\bar{x}\sim Exp(\theta=2000)$ (C) $\bar{x}\sim N(\mu=2000,\sigma=100)$

(D) $\bar{x}\sim N(\mu=2000,\sigma=400)$ (E) $\bar{x}\sim N(\mu=2000,\sigma=2000)$.

I-8.( ) Given that $x_i \overset{iid}{\sim} N(\mu,\sigma)$, which of the following statements about an unbiased estimator of parameters is correct?

(A) Unbiased estimator for parameter μ is unique.

(B) When the sample size increases, the variance of any unbiased estimator of μ should get smaller.

(C) $(n-1)S^2/n$ is an unbiased estimator for σ^2.

(D) $(x_1+x_2+x_3)/3$ is an unbiased estimator for μ.

(E) An unbiased estimator is automatically being a consistent estimator.

I-9.( ) Data were collected on the amount spent by customers for lunch at a restaurant in Tainan. The manager would like to find the confidence interval estimate of the mean amount spent for lunch by using $\bar{x}\pm z_{\alpha/2}\times(\sigma/\sqrt{n})$ (confidence level $1-\alpha$). Based upon past studies, the standard deviation is known with $\sigma=$ \$20. Which of the following experimental design would result in the shortest length the obtained confidence interval?

(A) $(n,1-\alpha)=(81,\ 95\%)$ (B) $(n,1-\alpha)=(81,\ 99\%)$ (C) $(n,1-\alpha)=(100,\ 99\%)$

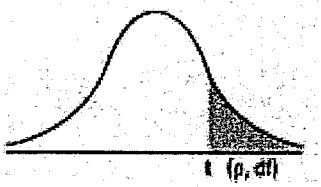
(D) $(n,1-\alpha)=(100,\ 95\%)$ (E) $(n,1-\alpha)=(100,\ 90\%)$.

I-10.( ) Following previous question I-9, if σ is unknown but the manager still wants to find the confidence interval estimate of the mean amount spent for lunch with confidence level $1-\alpha$. Based on the amount spent by 25 customers, the sample mean is \$250 and the sample variance $S^2=361$. Use the sample standard deviation to estimate σ, the confidence interval with 95% confidence level in this case is ?

(A) [100.9 399.0] (B) [210.7 289.2] (C) [243.7 256.2]

(D) [248.4 251.5] (E) [242.1 257.8]

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

Part II. Multiple Choice (50%)

Instructions: Draw a table like the following one on your answer sheets. The first row is for the number of questions. Fill your answers in the second row. Each question is 5%.

Question	1	2	3	4	5	6	7	8	9	10
Answer										

1. Which of the following statements is **not** correct?
   (A) The level of significance is the probability of making a Type I error when the null hypothesis is true.
   (B) We make a Type I error if we reject the null hypothesis when the null hypothesis is true.
   (C) We make a Type II error if we do not reject the null hypothesis when the alternative hypothesis is true.
   (D) If we collect more data, the probability of rejecting the null hypothesis decreases.
   (E) The sum of Type I and Type II errors is one when the null hypothesis is true.

2. The ABC company produces icewine, labeled as 375 ml on its bottle. The quality controllers of the company need to know if customers are getting less content than the statement. Suppose they are going to conduct a test of a hypothesis based on recently produced samples; they should use
   (A) a two tailed test.
   (B) a one tailed test with an alternative to the right.
   (C) a one tailed test with an alternative to the left.
   (D) either a one or a two tailed test is fine.
   (E) None of the above.

3. The life expectancy of a part produced by the current process resulting mean equals 100 hours and sample variance 81 from 9 samples. A new improved process has mean 106.48 hours and sample variance 84 from 12 samples. Suppose we perform a hypothesis test to see if the new process has a longer life expectancy than the current process; what is the p-value based on the observations?
   (A) 0.0526 (B) 0.1052 (C) 0.9474 (D) 0.8948 (E) None of the above.

4. A car tire manufacturer claims their premium car tires last longer than those produced by its biggest competitor. One experiment is designed to see if there is support for such a claim. Eight different luxury car models were selected for the test. Eight tires produced by the manufacturer were randomly installed on either the right or the left side of the front wheels of a car, and another eight tires were installed on the rear wheels using the same method. The rest were the competitor's tires. After each car had been driven 50,000 kilometers, the wear was measured. Which hypothesis test should they use for the experiment?
   (A) two sample z-test (B) two sample t-test (C) paired t-test (D) test for two proportions
   (E) completely randomized design ANOVA

Questions 5 – 7: Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon,\ \varepsilon \sim N(0, \sigma^2)$. You are given the following data: $\sum_{i=1}^{10} y_i = 1000,\ \sum_{i=1}^{10} x_i = 100,\ \sum_{i=1}^{10} x_i y_i = 10020,\ \sum_{i=1}^{10} x_i^2 = 1005,\ \sum_{i=1}^{10} y_i^2 = 100100$

5. The parameters to be estimated are:

   (A) β_0, β_1 (B) $\beta_0, \beta_1, \varepsilon$ (C) $\beta_0, \beta_1, 0$ (D) $\beta_0, \beta_1, \sigma^2$ (E) β_0, β_1, x

6. What is the ordinary least squares estimate of $\hat{\beta}_1$?

   (A) 4 (B) 4.25 (C) 4.5 (D) 4.75 (E) None of the above

7. What is the R^2?

   (A) 0.7 (B) 0.75 (C) 0.82 (D) 0.85 (E) None of the above

Questions 8 – 10: The strength of the material is determined by two factors: A and B. There are three levels for Factor A and four levels for Factor B. The balanced two-way ANOVA table of the experiment is given as the following one with some missing values. The response of strength is y. We have $\sum\sum\sum y_{ijk}^2 = 662$, and the average of $y = 2$. Complete the ANOVA table and answer the following questions. Note that the F-table is not provided in the exam.

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-value
A	44			11
B	90			15
Interaction	SSAB			
Error	96			
Total	SST	59		

8. What is the value of SST in the table?

   (A) 418 (B) 422 (C) 658 (D) 662 (E) None of the above

9. What is the value of SSAB in the table?

   (A) 168 (B) 192 (C) 218 (D) 242 (E) None of the above

10. Which of the following conclusions is true given $\alpha = 0.05$?

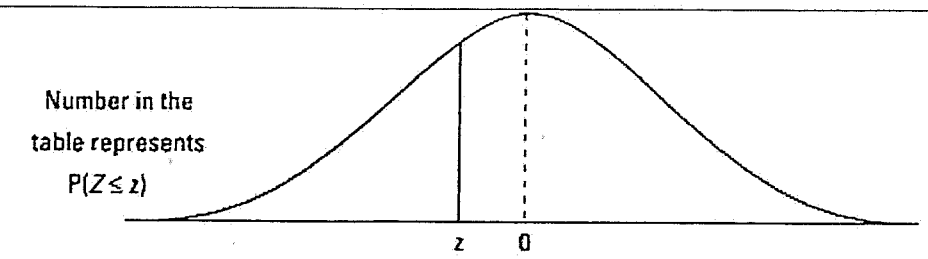
   (A) Factor A, Factor B, and interaction are all significant.

   (B) Factor A and Factor B are significant. Interaction is not significant.

   (C) Only Factor B and interaction are significant.

   (D) Only interaction is significant.

   (E) Cannot be determined without the F-table.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641