國立成功大學
115學年度碩士班招生考試試題

編　號：158

系　所：工業與資訊管理學系

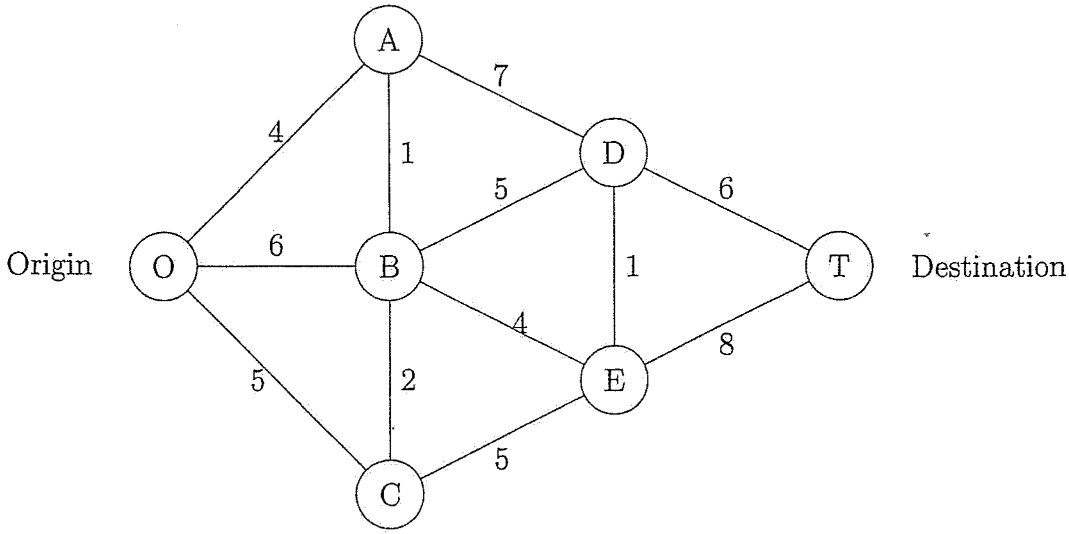
科　目：作業研究

日　期：0203

節　次：第 2 節

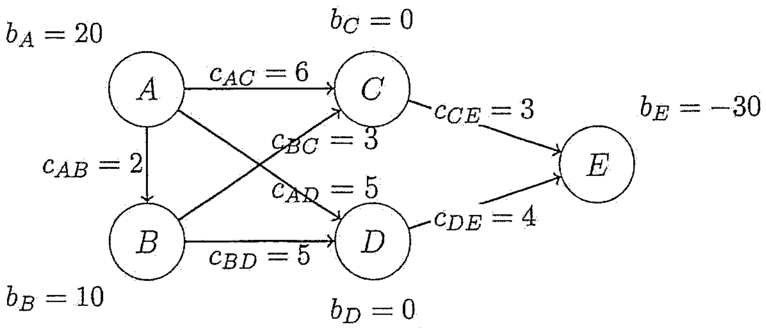
注　意：1.可使用計算機
　　　　2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. [10%] Consider the undirected and connected network with two special nodes called the origin and the destination. Associated with each of undirected arcs is a nonnegative distance between the pair of nodes connected by that arc.



Use the dynamic programming to determine the shortest route from Node O to Node T.

2. Consider the minimum cost flow problem shown below, where the b_i values (net flows generated) are given by the nodes, the c_{ij} values (costs per unit flow) are given by the arcs, and the u_{ij} values (arc capacities) are given between nodes.



Arc capacities:
$u_{AC} = 10$
$u_{BC} = 25$
Others: ∞

(a) [5%] Formulate the give minimum cost flow problem as a linear programming problem.

(b) [5%] Consider the following flow:

Arc	Flow
$A \to D$	0
$B \to D$	0
$D \to E$	0

and find the flow on the other arcs in order to produce a feasible flow for the minimum cost flow problem. Is the feasible flow a basic solution to the problem in (a)?

(c) [5%] Verify that the solution in (b) is optimal.

3. Consider the maximization linear programming problem

$$
\begin{aligned}
\max\quad & c_1x_2 + c_2x_2 + c_3x_3 \\
\text{s.t.}\quad & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + x_4 = b_1 \\
& a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + x_5 = b_2 \\
& a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + x_6 = b_3 \\
& x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}
$$

and the optimal simplex tableau is given by

$B.V.$	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	0	0	0	3	0	5	θ
x_1	0	1	1	0	2	0	1	2
x_3	0	0	0	1	1	0	4	2
x_5	0	0	-2	0	-1	1	3	1

(a) [5%] Obtain the optimal value from the tableau.

(b) [5%] Another firm wishes to purchase one unit of the first resource b_1 from you. How much is such a unit worth to you? Why?

(c) [5%] Are there any alternative optimal solutions? If not, why not? If so, give one.

4. Consider the following linear programming problem:

$$
(P)\qquad
\begin{aligned}
\max\quad & 4y_1 + 6y_2 \\
\text{s.t.}\quad & y_1 - y_2 \leq 2 \\
& 4y_1 + 5y_2 \leq -3 \\
& -2y_1 - y_2 \leq 1 \\
& y_1 \in \mathbb{R}, y_2 \leq 0
\end{aligned}
$$

(a) [5%] Write down the KKT condition for the problem (P)

(b) [5%] Directly explain why the KKT condition in (b) can produce an optimal solution of the problem (P).

5. (15%) In a large-scale optimization model, the production quantity x appears as a decision variable determined by other constraints and objectives. The production cost function associated with x is given by the following piecewise linear function:

$$C(x) = \begin{cases} 5x, & 0 \leq x < 100 \\ 500 + 8(x - 100), & 100 \leq x \leq 180 \\ 1140 + 12(x - 180), & x > 180 \end{cases}$$

Formulate a **mixed-integer linear programming (MILP)** representation of the cost function $C(x)$. Your formulation should:

1. Introduce appropriate auxiliary variables and binary variables,
2. Ensure the correct activation of cost segments,
3. Be **exact** (i.e., no approximation),
4. Be linear and suitable for standard MILP solvers.

You **do not** need to optimize over x, and no additional system constraints are required.

6. (15%) Consider a **two-player zero-sum game** between Player A and Player B. The payoff matrix below shows **Player A's payoff**.

A \ B	B_1	B_2	B_3	B_4
A_1	5	1	4	0
A_2	6	2	5	1
A_3	1	4	2	3

Solve the game **using the graphical method**. Determine:

(a) (5%) The optimal mixed strategy of Player A,

(b) (5%) The optimal mixed strategy of Player B, and

(c) (5%) The value of the game.

7. (20%) A person commutes between home and office, making one trip at a time. Each trip is independent, and it rains with probability p. The person owns two umbrellas in total.

1. If it rains and at least one umbrella is available at the departure location, the person takes one umbrella to the destination.
2. If it rains and no umbrella is available at the departure location, the person gets wet.
3. If it does not rain, no umbrella is moved.
4. At most one umbrella can be carried per trip.

The system is observed immediately before each trip.

Questions:

(a) (10%) Model the system as a Markov chain and construct the transition probability matrix.

(b) (5%) Find the stationary distribution.

(c) (5%) In the long run, what fraction of trips does the person get wet?