國立成功大學

115學年度碩士班招生考試試題

編　號：160

系　所：工業與資訊管理學系

科　目：統計學

日　期：0203

節　次：第 3 節

注　意：1.可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1、The president of a bank believes that the variance of the deposits of suburban customers is more than the variance of city customers. Below you are given the results of samples taken from suburban and city customers.

	Suburban Customers	City Customers
Sample Size	21	19
Sample Mean	520	640
Sample Variance	150	90

(1) State the null hypothesis for testing and justify. Please clearly define your symbols. (6%)

(2) Compute the value of the test statistic (2%) and specify its degrees of freedom (2%).

(3) State the necessary assumptions to perform this test. (5%)

2、In applying the simple random sampling on an infinite population, argue why a larger sample size tends to provide point estimates closer to the population mean. (10%)

3、Let a categorical variable have three possible value A, B, and C. When it is replaced by the dummy variables x_1 and x_2 given below, the estimated linear regression equation is $\hat{y} = 7.2 + 1.9x_1 - 0.3x_2$. Give the linear regression equation when the categorical variable is replaced by x_1' and x_2'. (10%)

Dummy	x_1	x_2	x_1'	x_2'
A	0	0	1	0
B	1	1	0	1
C	0	1	1	1

4、A group of 500 individuals were asked to cast their votes regarding a particular issue of the Equal Rights Amendment. The following table shows the results of the votes.

Sex	Favor	Undecided	Oppose	TOTAL
Female	180	80	40	300
Male	150	20	30	200
TOTAL	330	100	70	500

(1) Define random variables to formulate the null hypothesis for testing whether voter preference is independent of the sex of the individuals. (3%)

(2) Compute the value of the test statistic (8%) and clearly specify its degrees of freedom. (2%)

(3) Argue why the test can be applied on this application (2%).

5. (10%) The joint probability density function of X and Y is given by

$$f(x,y)=\frac{8}{5}\left(x+\frac{xy}{2}\right) \quad 0<x<1, 0<y<1$$

(a) (5%) Find E[X]

(b) (5%) Find P{$X > Y$}

6. (20%) Let $Y \sim$ Uniform (0, T). First, draw a value of Y. Given this value of Y, observe a random variable Z whose distribution is Poisson with parameter λY, where $\lambda>0$ and $T>0$ are constants.

(a) (10%) Find E[Z].

(b) (10%) Find Var[Z].

7. (20%) Let (X, Y_1, Y_2) be a three-dimensional random vector with finite second moments. Suppose that $E[X]=E[Y_1]=E[Y_2]=0$, Var[X]=4, Var[Y_1]=Var[Y_2]=9, Corr(X,Y_1)=ρ_1, Corr(X,Y_2)=ρ_2, and Corr(Y_1,Y_2)=r. Define T= Y_1+ Y_2. Among all linear predictor of T based on X of the form: g(X)=a+bX, answer the following questions.

(a) (10%) Find the values a*, b* that minimize the mean squared error: E[(T-g(X))^2].

(b) (10%) Find the minimum mean squared error: $\min_{a,b}$ E[(T-g(X))^2].