國立成功大學75學年度工業管理研究所考試(試題) 共 1 頁 第 1 頁

1. For a region defined by $F=\{(x_1\ x_2\ x_3)|-2x_1+2x_2+x_3\leq 5,\ 3x_1+x_2-x_3\leq 10,\ x_1,x_2,x_3\geq 0\}$ Please find all of the extreme point of F. (15%)

2. Formulate the dual of the following linear programming problem:

$$\max.\ c_1x_1+c_2x_2+\ldots+c_nx_n$$

$$\text{s.t. } a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n\leq b_1$$
$$a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n\leq b_2$$
$$\vdots$$
$$a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n\leq b_m$$
$$d_i\leq x_i\leq e_i,\ i=1,2,\ldots,n$$

or in matrix form:

$$\max.\ c'x$$
$$\text{s.t. } Ax\leq b$$
$$d\leq x\leq e$$

where a_{ij}, b_i, c_i, d_i, and e_i are real numbers. (20%)

3. For a queueing system with infinite servers, the interarrival times are exponentially distributed with parameter λ and the service time of every server is also exponential but with parameter μ, i.e., we have a M/M/∞ queueing system. Please derive the expected number of persons in the system (L), in the queue (L_q), and the expected waiting time in the system (W) and in the queue (W_q). (20%)

4. For the simplest EOQ (Economic Ordering Quantity) model in inventory theory, the optimal ordering quantity Q* is calculated as:

$$Q^*=\sqrt{2aK/h}$$

where a is the demand per unit time, K is the setup cost, and h is the inventory holding cost per item per unit of time. If the quantity ordered is 100% more than Q*, i.e., Q=2Q*, then how many percent will the total cost increase correspondingly? The cost of items is ignored. (Hint: You have to find the equation of total cost first.) (20%)

5. Please derive the Chapman-Kolmogorov equations, i.e.,

$$p_{ij}^{m+n}=\sum_{k=0}^{\infty}p_{ik}^{n}p_{kj}^{m}\quad \text{for all } n,m\geq 0,\ \text{all } i,j.$$

where $p_{ij}^{n}=Pr\{x_{n+s}=j|x_s=i\}$, $n\geq 0$. (10%)

6. Explain what is (i) recurrent state, (ii) communicating class, and (iii) aperiodic state in Markov chains. Also explain shortly what is policy-improvement and value-improvement in Markov decision processes. (15%)