國立成功大學75學年度工業管理研究所考試(應用數學試題) 共1頁 第1頁

1. Find the derivative of $f(x)=x^{x^2}\cdot e^{x^3}$. (10%)

2. Find $\int_0^1 x^{-0.2}(1-x)^{-0.4}dx$. (10%)

3. Show that $\sum_{i=0}^{n}(-1)^i\binom{n}{i}=0$, where $\binom{n}{i}=\frac{n(n-1)\ldots(n-i+1)}{i(i-1)\ldots\ldots..1}$. (10%)

4. Let A be an nxn orthogonal matrix, i.e., $A^{-1}=A'$. Please show that A is nonsingular, furthermore, its determinant is equal to either +1 or -1. (10%)

5. Show that (15%):

(i) $\binom{-n}{k}=(-1)^k\binom{n+k-1}{k}$

(ii) $\sum_{j=0}^{\infty}\binom{n+j-1}{j}x^j=(1-x)^{-n}$

(iii) $\sum_{j=0}^{\infty}(j+1)x^j=\frac{1}{(1-x)^2}$

6. Find the Laplace transform of $f(x)=(1-e^{-x})/x$. (Hint: The Laplace transform of a function f(x) is defined as $L(f(x))=\int_0^{\infty}e^{-tx}f(x)dx$.) (15%)

7. An nxn matrix A is an idempotent matrix if it satisfies (i) $A=A'$ and (ii) $A=A^2$. Now, if the rank of A is k, show that A has k nonzero characteristic roots and they are each equal to +1. (15%)

8. Find the determinant and inverse of the matrix A, where

$$A=\begin{bmatrix} 0 & I & I & \ldots & I \\ I & 0 & I & \ldots & I \\ I & I & 0 & \ldots & I \\ & & \vdots & & \\ I & I & I & \ldots & 0 \end{bmatrix}$$

each identity has size nxn, and there are k^2 submatrices. (15%)