工業管理研究所

1. Find $\int_0^1 \ln(x)dx$. (12%)

2. Calculate $\sum_{k=0}^{\infty}\int_0^1((-1)^k x^k/k!)dx$. (12%)

3. What is basis? Show that the vectors (1 0 3), (2 1 7), and (0 0 3) form a basis for the 3-dimensional space. (12%)

4. Prove that the necessary condition for x* to be a local extremal point of a continuous function (differentiable) f(x) is $\nabla f(x^*)=0$. (12%)

5. Suggest one method for solving a set of m nonlinear equations with n variables. (12%)

6. For the following set of differential equations

$$dx_0(t)/dt=-\theta x_0(t)$$

$$dx_n(t)/dt=-\theta x_n(t)+\theta x_{n-1}(t), \quad n=1,2,3,...$$

with boundary conditions

$$\lim_{t\to 0} x_0(t)=1$$

$$\lim_{t\to 0} x_n(t)=0, \quad n=1,2,3,...$$

Solve $x_n(t)$, n=0,1,2,... (20%)

7. A quadratic function $f(x)=3x_1^2+4x_1x_2+6x_2^2$ in matrix form is $f(x)=x'Ax$, where A is symmetric. This quadratic function can be transformed to $g(y)=y'Q'AQy$, where $x=Qy$ and $D=Q'AQ$ is a diagonal matrix. Please find A, Q, and D, also show that the diagonal elements of D are just the eigenvalues of A. (20%)