

1. For the matrix given below, find A^{-1} ; also give the conditions such that A^{-1} exists. (10%)

$$A = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$

2. Show that $\int_0^1 x^{a-1}(1-x)^{b-1}dx = \int_0^1 x^{b-1}(1-x)^{a-1}dx$, $a, b > 0$. (10%)
3. Show that $e^{ix} = \cos(x) + i \cdot \sin(x)$, where $i = \sqrt{-1}$. (10%)
4. Find $\frac{d}{dy} \int_y^{y+1} (x-y)^2 e^{-x} dx$. (10%)
5. Show that the vectors of an orthogonal matrix are independent. Give an example for the 2x2 matrix. (Hint: a matrix A is orthogonal if $A^T A = I$). (10%)
6. The convex hull of a finite number of points x_1, \dots, x_k , is the set of all convex combinations of x_1, \dots, x_k . The convex hull of any set of $n+1$ points from E^n which do not lie on a hyperplane in E^n is called a simplex. In the followings, which sets of points constitute a simplex? Give your reason. (10%)
- (a) $(1, 1), (1, 4), (4, 1)$
- (b) $(1, 1, 1), (2, 2, 2), (1, 0, 1)$
- (c) $(1, 1, 1), (2, 2, 2), (1, 0, 1), (4, 3, 4)$
7. A function $f(x)$ is defined as "o(x)" function if $\lim_{x \rightarrow 0} f(x)/x = 0$.
- (a) Is $f(x) = e^{-x}$ a o(x) function?
- (b) Suppose $f(x)$ and $g(x)$ are o(x) functions, is $h(x) = f(x) \cdot g(x)$ a o(x) function? (10%)
8. Which of the following sets are convex? Explain why. (15%)
- (a) $S = \{(x_1, \dots, x_n) \mid x_1 + x_2 + \dots + x_n = 1\}$
- (b) $S = \{(x_1, \dots, x_n) \mid x_1 \leq x_2 \leq \dots \leq x_n\}$
9. (a) Find the area of the region bounded by $y = x^2$ and $y = 4 - x^2$.
- (b) Find the volume of the solid under the surface $z = 4 - x^2 - 4y^2$ and above the region of the xy plane bounded by $x = 0, y = 0$, and $x + 2y - 2 = 0$. (15%)