

**Instructions:**

1. No discussion is allowed before you hand in.
2. You may use a calculator, if it is necessary.
3. Partial credits will be given only to work that is clear and is understandable. Please answer the problems sequentially.
4. Good luck!

**Problems:**

1. One of the tough questions in inventory decisions is to decide the order quantity  $Q$  so that the expected profit can be maximized. Let's define  $c$  as the cost per item,  $s$  as the selling price per item,  $v$  as the salvage value per item,  $p$  as the lost sales penalty, and  $f(x)$  as a density function of the demand. Now, if a *critical ratio* policy is derived as follows:

$$\int_Q^{\infty} f(x)dx = \frac{c-v}{s+p-v}$$

and we know that the demand is uniformly distributed over the range  $[a, b]$ . We therefore can derive an explicit, closed-form expression for  $Q =$  \_\_\_\_\_ . (25 points)

2. A quality control engineer is puzzled by the measurement error during the manufacturing of a rectangle. If he has found out that the rectangle has dimensions of 2.5 and 4.5 millimeters, with an error of at most 0.01 millimeter in each measurement. The maximum error that might occur if these numbers are used to compute the area of the rectangle is \_\_\_\_\_. (15 points)
3. Please write an algorithm to implement a function that can calculate the natural logarithm of  $x$ . If  $x = 0.97$ , the answer solved by this algorithm and represented by scientific notation, given mantissa is 5 digits, should be \_\_\_\_\_. (20 points)
4. On a certain route (e.g., Tainan to Taipei), a train carries 7500 passengers per month, each paying \$230. If the manager wants to increase the fare, while the market research department estimates that for each \$1 increase in fare, the train will lose 100 passengers. The price that maximizes the train's revenue is \_\_\_\_\_. (15 points)
5. Suppose that  $x$  units of labor and  $y$  units of capital can produce  $f(x, y) = 50x^{0.5}y^{0.25}$  units of a certain product. Assuming that each unit of labor costs \$150, each unit of capital costs \$100, and \$25,000 is available to spend on production. You may find out that the manager should utilize \_\_\_\_\_ units of labor and \_\_\_\_\_ units of capital to maximize production. (25 points)