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國立成功大學 79 學年度 工管所 考試(管理數學 試題) 共 1 頁 第 1 頁

1. Apply the Newton-Raphson method to find the extreme points of the following function: (10%)

   $f(x)=4x^4-3x^2+3$

2. Two firms are in competition, each having profit depending in part upon the output of the other. Let the outputs of the two firms be O_1 and O_2, while their profits are P_1 and P_2. Suppose these quantities are related by the equations (15%)

   $P_1=70\ O_1-O_1^2-O_2^2$ $\qquad$ $P_2=90\ O_2-2\ O_2^2-\frac{1}{2}O_1^2$

   Determine profits if each firm acts independently to maximize its profit. Then determine the maximun value of P_1+P_2 if the firms decide to cooperate.

3. (i) Solve the difference equation given below, and then find r(7). (10%)

   $r(n)=r(n-1)+2r(n-2); r(1)=2$ and $r(2)=12$

   (ii) Write the difference equation $r(n)=r(n-1)+2r(n-2)$ in matrix form $R(n+1)=AR(n)$, and then find the eigenvalues and eigenvectors of A. (10%)

4. Solve the following problem graphically: (10%)

   maximize $z=\min\{2x-8, -5x+3\}$
   subject to

   $0\leq x\leq 4$

5. Find $\int x^3\sinh(x)\,dx$ (10%)

6. For (15%)

   $$A=\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{pmatrix}$$

   the eigenvalues of A are -1, -2, -3. There is a matrix P such that

   $P^{-1}AP=D$ is diagonal. Find P, D.

7. Find the standard matrices for linear transformations T and T+U. (10%)

   $T(x,y)=(x-y,x)$, $\quad U(x,y)=(2y,x)$

8. Prove that (10%)

   $$\sum_{n=1}^{\infty}\frac{1}{(n+2)n!}=\frac{1}{2}$$