

1. The owner of a chain of three grocery stores has purchased five crates of fresh strawberries. The estimated probability distribution of potential sales of the strawberries before spoilage differs among the three stores. Therefore, the owner wants to know how he should allocate the five crates to the three stores to maximize expected profit. (15%)

No. of crates	Stores		
	1	2	3
0	0	0	0
1	5	6	4
2	9	11	9
3	14	15	13
4	17	19	18
5	21	22	20

For administrative reasons, the owner does not wish to split crates between stores. However, he is willing to distribute zero crates to any of his stores. The table gives the estimated expected profit at each store when it is allocated various numbers of crates.

Use dynamic programming to determine how many of the five crates should be assigned to each of the three stores to maximize the total expected profit.

2. Solve the following bounded variable LP. (15%)

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$-z$	$b$
1	0	0	1	2	-1	1	0	13
0	1	0	-1	1	1	2	0	9
0	0	1	2	2	2	-1	0	5
3	4	-2	-5	3	2	-1	1	0

$0 \leq x_j \leq 5$  for all  $j$ ,  $z$  to be minimized

3. Six different items are being considered for shipment. The weight and value of each item is given in the right table.

Item	weight	value
1	10	5
2	9	2
3	15	7
4	2	4
5	11	1
6	6	6

Formulate and solve a 0-1 integer program whose solution will give the shipment of maximum value when the total weight of the shipment can be no larger than 33. (15%)

4. Solve the following nonlinear programming problem by Kuhn-Tucker conditions

Min.  $f(x) = \ln(x_1 - x_2)$

s.t.  $g_1(x) = -x_1 \leq 0$

$g_2(x) = -x_1 + x_2 + 1 \leq 0$

$g_3(x) = x_1^2 + x_2^2 - 16 \leq 0$

(20%)

5. Trucks arrive at a safety inspection station so that interarrival times are exponentially distributed with mean of  $\frac{1}{2}$  hour. The times required for inspection are also exponentially distributed with mean of  $\frac{1}{5}$  hour. Assume the associated queuing is in steady state.

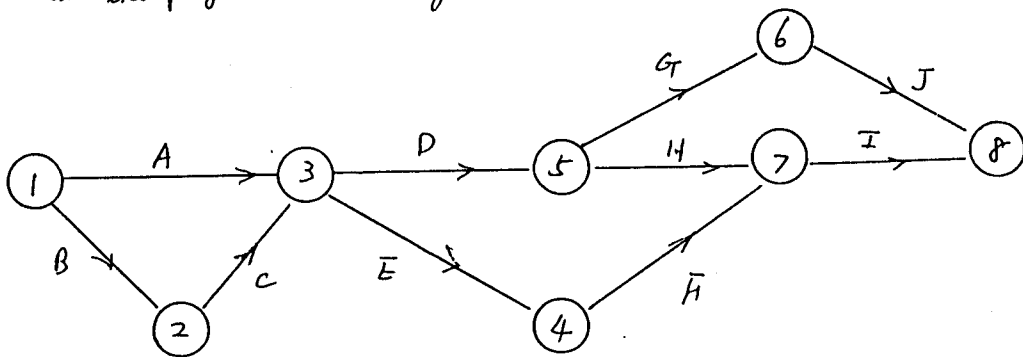
(a) Construct the rate diagram for this queuing system.

(b) Calculate  $P_0$  and then  $P_n$  for  $n \geq 1$  (15%)

(c) calculate  $L$

(d) Given  $L$ , calculate  $w$ ,  $L_q$  and  $w_q$ .

6. Consider the project network given below:



The data for normal times, crash times and crashing costs are given as follows:

follows:

Let  $T$  represent the earliest completion time of the project.

(a) Determine the maximum and the minimum value of  $T$

(b) Determine the critical path

(c) Given the overhead costs as \$5 per day, determine the optimal duration of the project in terms of both the crashing and the overhead costs.

Job	Normal time (days)	Crash time (days)	Cost of crashing per day (\$)
A	10	7	4
B	5	4	2
C	3	2	2
D	4	3	3
E	5	3	3
F	6	3	5
G	5	2	1
H	6	4	4
I	6	4	3
J	4	3	3

(20%)