1. Write down the definition of the following terms. (10%)
   (a) directional derivative (b) implicit differentiation (c) Jacobian
   (e) leading principal minor (d) positive definite

2. Calculate $\int_0^2 [\max(3x,4-x^2)-\min(3x,4-x^2)]\,dx$. (10%)

3. Let f ' be continuous in [a, x]. State under what conditions (10%)
   $D[\int_a^x f(t)\,dt] = \int_a^x (Df)(t)\,dt$.

4. Evaluate the integral $\int dx/[(x)(\ln x)(\ln \ln x)]$. (10%)

5. Find the derivative of $f(x)=x^{x^x}$ (10%)

6. Find the sum of the series $S(x)=\sum_{n=2}^{\infty} x^n/[(n-1)n]$. (10%)

7. Suppose a matrix A can be partitioned as on the right, where a: scalar, b: 1xn, 0: nx1 null vector, C: nxn. Find A^{-1} in terms of a, b, C and C^{-1}. (10%)
   $A = \begin{bmatrix} a & b \\ 0 & C \end{bmatrix}$

8. A square matrix A which satisfies A'A=I is called an orthogonal matrix. If we denote $A=\|a_{ij}\|$, find the determinant of A. (7%)

9. A symmetric matrix A which satisfies AA=A is called idempotent. For example, in regression, the residual: $e=[I-X(X'X)^{-1}X']Y=[I-A]Y$, A is idempotent, where $A=X(X'X)^{-1}X'$. Show that A is indeed idempotent. Also show that if A is idempotent then [I-A] is also idempotent as is the case in the above example. (8%)

10. Let f(x,y)=1/x+2/y. Use (a) linear, (b) quadratic, (c) cubic approximation, respectively to approximate f(x,y) at the point [1 1]. (15%)