

1. 已知 $xy^2 = \sin(x+2y)$, 求 $\frac{dy}{dx}$, (5%)
2. 設 $f(x) = \frac{1-\sqrt{x}}{1+\sqrt{x}}$, $x > 0$, 求 $Df(x)$, $D^2f(x)$, (6%)
3. 求 $\sum_{n=1}^{\infty} n(1-p)p^n$, $0 < p < 1$ (5%) 4. 求 $\int \sqrt{x} \ln x \, dx$ (5%)
5. 求 $\int \frac{8x^3+7}{(x+1)(2x+1)^3} \, dx$ (6%) 6. 求 $\int_{-\infty}^{\infty} e^{-x^2} \, dx$ (6%) 7. $\int \frac{1}{\sin x + \cos x} \, dx$ (6%)
8. 試證 $\int_x^1 \frac{dt}{1+t^2} = \int_1^x \frac{dt}{1+t^2}$, $x > 0$, (5%)
9. 已知 $f(x) = \int_0^x \frac{t^2(1-2t)}{1+t^6} \, dt$, 試求 $f(x)$ 在何處發生極值, 極大抑極小? (6%)

10. Denote $x=[x_1, x_2, \dots, x_n]$ and $y=[y_1, y_2, \dots, y_n]$, show that the cosine (i.e., $\cos \theta$) of the angle between x and y is the correlation coefficient of x and y . (10%)

11. If the n -component vectors a , b , and c are linear independent, show that $a+b$, $b+c$, and $c+a$ are also linear independent. (10%)

12. What is positive definite? For the matrix A on the right, within what ranges of α , β , γ will A and its associated quadratic form $x'Ax$ be positive definite? (10%)

$$A = \begin{bmatrix} 2+\alpha & 1 \\ 1-\beta & \gamma \end{bmatrix}$$

13. For an $M/M/\infty$ queueing system, suppose the arrival rate is λ , service rate is μ , and there are infinite servers. Derive the average number of customers in the system. (15%)

14. Derive the well-known economic lot-size formula (also called EOQ formula, Willson formula, or Harris formula) in inventory theory. (5%)