1. Short Problems (30%)
   (a) Find $\lim_{x\to\infty} x^{1/x}$
   (b) Find $\lim_{x\to 0} (1-x)^{1/x}$
   (c) Find $\lim_{x\to 1} (\sum^{n}_{k=1} x^{k}-n)/(x-1)$ (Give a closed-form result)
   (d) Find $\lim_{x\to 0} [(4+x+x^2)^{1/2}-2]/[(4+x-x^2)^{1/2}-2]$
   (e) Find $[\sum^{\infty}_{x=1} x/(x+1)!]$
   (f) Find $\nabla f(x,y)=(\pi+\tan^{-1}x)^{y}$

2. Find the distance between the point (d,e,f) and the plane ax+by+cz=0. (10%)

3. Find the length, width, and height of a rectangular box without top to have a given volume of one unit so as to use the least amount of material. (10%)

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4. Find the following indefinite integrals:

(1) $\int \frac{t}{(1+t)^2}\, dt$ (6%)

(2) $\int \frac{w+2}{w-3}\, dw$ (6%)

5. Find the area bounded by the graphs of the functions
$f(x) = x^3 - 3x^2 + 2x$ and $g(x) = -x^3 + 4x^2 - 3x$ (8%)

6. Prove

if $A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots\ a_{1,m} \\ 0 & a_{2,2} & \cdots\ a_{2,m} \\ \vdots & \vdots & \vdots \\ 0 & 0 & \cdots\ a_{m,m} \end{pmatrix}$

then,

$|A| = a_{1,1}\, a_{2,2} \cdots a_{m,m}$ (10%)

7. Consider the polyhedron $P = \{x \in R^n \mid Ax = b, x \geq 0\}$.

Prove: If $b \geq 0$ and A (or AP where P is some permutation matrix) can be partitioned as A = [B ; N]
where B is nonsingular and all elements of B^{-1} and N are nonnegative,

then P is nonempty and bounded. (12%)

Note: $B^{-1} \geq 0 \not\Rightarrow B \geq 0$ For example for

$$B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix},\ B^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

8. Show that a hyperplane $H = \{\, \mathbf{x} : \mathbf{px} = k\}$ and a halfspace $H^+ = \{\, \mathbf{x} : \mathbf{px} \geq k\,\}$ are convex sets. (8%)