

一、一袋中有仟元鈔一張，伍佰元鈔五張，壹佰元鈔四張。某人自此袋中以下列方式隨機抽取 (i) 一次抽二張，(ii) 抽一張登記後放入再抽取一張，如此方式抽取二張。試分別求此抽取之期望值 (Expected Value) 為多少？

15%

二、為了估計某位立委候選人得票率作民意調查，現以隨機抽樣法抽出樣本數 n 的一組隨機樣本。設 X 為樣本中贊成投他的票數，以樣本比率：

$$\hat{p} = \frac{X}{n} \quad \text{估計真值 } P \text{ 值 } (P \text{ 為母體比率})。$$

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(1) 現規定估計誤差不大於 0.05，信賴度 $1 - \alpha = 0.95$ ，問抽樣數至少為多少？

(2) 根據 (1) 所得之抽樣數，按簡單隨機抽樣法 (Simple random sampling) 實際抽得 $X=38$ ，求該位候選人真正得票率在信賴度是 0.95 下，落在多少百分比之間？

三、設從已知變異數 (Variance) 是 9，但平均數 (Mean) μ 未知的常態分配中隨機抽出 n 個樣本 X_1, X_2, \dots, X_n ，在顯著水準 $\alpha = 0.05$ 下，欲檢定假設 $H_0: \mu = 15$ 對假設 $H_1: \mu > 15$ 。

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(1) 寫出決策規則 (即寫出棄卻域 Reject region 的形式)

(2) 若樣本數 $n=36$ ，求 $\mu = 16$ 時的檢定力 (Power of test)

(3) 若希望 $\mu = 16$ 時的檢定力達到 0.95，問至少需取幾個樣本。

四、設有 N 人出席會議，每人名牌上號碼依次為 $1, 2, \dots, N$ ，為了估計與會人數 N ，隨機抽取 5 張名牌，其號碼分別為 $\{37, 16, 44, 43, 22\}$ 。試以動差法 (Method of moment) 估計 N ，並問所求之估計式 (Estimator) 滿足不偏性 (Unbiased estimator) 嗎？理由，加以證明之。

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後附統計用表

五. 一線性迴歸模式(Linear Regression Model)以矩陣表示如下:

$$Y = X\beta + \varepsilon$$

其中

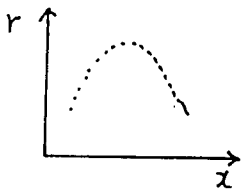
$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}_{n \times 1}, \quad X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1k} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ 1 & x_{n1} & \cdots & x_{nk} \end{bmatrix}_{n \times (k+1)}, \quad \beta = \begin{bmatrix} \beta_0 \\ \vdots \\ \beta_k \end{bmatrix}_{(k+1) \times 1}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}_{n \times 1}$$

令 B 為 β 以最小平方方法(Least Square Methods)所產生之不偏估計值矩陣,

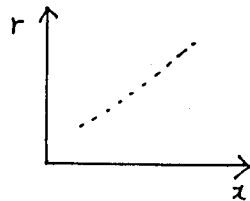
\hat{Y} 為預測值(predicted value)矩陣, r 為殘值(residual)矩陣。

試證明下列(1)~(3)題(其中 A' 代表 A 之轉置(transpose)矩陣)

- (1) $1'r = 0$ (5%) ($1' = [1 \cdots 1]_{1 \times n}$)
- (2) $X'r = 0$ (5%) ($0' = [0 \cdots 0]_{1 \times (k+1)}$)
- (3) $\hat{Y}'r = 0$ (10%)
- (4) 若 R^2 (Coefficient of determination) = 0.80, 請說明其代表的意義。(5%)
- (5) 以下二圖為利用簡單線性迴歸模式($y_i = b_0 + b_1 x_i$)所得之殘值與自變數 x_i 之關係圖, 各說明其可能之現象?(5%)



(圖一)



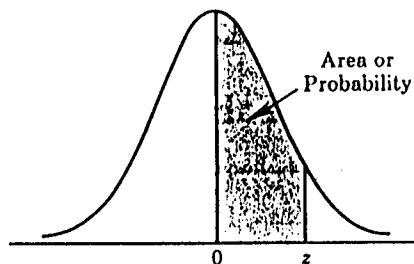
(圖二)

- 六. (1) 利用因子設計(Factorial Design)作實驗有何優點?(5%)
(2) 某 2^k ($k=2$) 之因子設計如下表(8%)

		Factor B	
		Low level	High level
Factor A	Low level	30	60
	High level	60	30

- (a) 求A,B之效果(effect)
(b) A與B對實驗之反應值(response)是否有影響效果?說明其理由。
- (3) 若欲將5個因子(A, B, C, D, E)之 2^5 實驗,分成4個blocks,做"1/4 fraction"之部份因子設計(Fractional Factorial Design)時,以下二個劃分標準(defining relation),那一個較佳?說明其理由。(7%)
- (a) $I=ABCDE=BCDE$
(b) $I=ABC=BCDE$

TABLE Standard Normal Distribution



Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean and z is .3944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990