1. (10 points) Find

$$\int_1^7 \frac{(\ln x)^3}{x}dx$$

$$\int_0^1 \frac{2x}{(x^2+1)^3}dx$$

2. (5 points) Solve $y' = \frac{te^t}{y}$, $y(0) = -5$.

3. (10 points) Find

$$\lim_{x \to 0} \frac{\sqrt{9+x}-3}{x}$$

$$\lim_{x \to \infty}(1+\frac{1}{2x})^x$$

4. (5 points) Find the point at which $f(x,y,z) = 2x^2+3y^2+z^2-2x-y-z$ has minumum value.

5. (5 points) Write $x_1^2 + x_2^2 - 3x_3^2 + 2x_1x_2 - 4x_2x_3 + 6x_1x_3$ as a matrix product.

6. (5 points) Find the transition matrix representing the change of coordinates on P_3 from the ordered basis $[1, x, x^2]$ to the ordered basis $[1, 1+x, 1+x+x^2]$.

7. (5 points) Determine $\| \cdot \|_F, \| \cdot \|_\infty, \| \cdot \|_1$ for the following matrix.

$$\begin{bmatrix} 0 & 5 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

8. (5 points) Show that any 3x3 matrix of the form

$$\begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & b \end{bmatrix}$$

is defective.

9. (10 points) Prove by mathematical induction that

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

10. (8 points) Find

$$\int (x+3)e^{x^2+6x} dx$$

11. (8 points) Solve $y' = e^{-y}$, $y(0) = 0$.

12. (8 points) $f(x,y) = e^{\frac{x}{y}}$ Find $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$.

13. (8 points) Solve the following system of equations

$$
\begin{array}{rcrcrcrcl}
x_1 & + & x_2 & + & 2x_3 & + & x_4 & = & 8 \\
x_1 & & & + & x_3 & + & x_4 & = & 6 \\
 & & x_2 & & & + & x_4 & = & 4 \\
x_1 & - & x_2 & - & x_3 & + & x_4 & = & 6
\end{array}
$$

14. (8 points) Find a matrix P such that $P^{-1}AP$ is diagonal, where $A = \begin{bmatrix} 7 & 4 \\ -2 & -2 \end{bmatrix}$