1. Find $\lim_{y \to 1} \frac{\sqrt{y}-1}{y-1}$. (5%)

2. Find the point of the graph of the equation $y = x^2$ that is nearst the point A(3, 0). (10%)

3. Find $\int e^x \cos x \, dx$. (5%) , Find $\int \frac{x^3}{\sqrt{1+x^2}} \, dx$. (5%)

4. Find $\lim_{h \to 0^+} (1+ah)^{\frac{1}{h}}$. (10%)

5. Find the volume of the solid bounded above by the paraboloid $z = 4 - x^2 - y^2$ and below by the plane $z = 4 - 2x$. (15%)

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PART II (Linear Algebra):

6. (10%) Give a definition for the following terms: a). Convex set, b). Hyperplane, c). Positive semi-definite, and d). Rank of a matrix.

7. (10%) Let

$$A = \begin{vmatrix} B & 0 \\ T & I \end{vmatrix}$$

Where B is an m x m invertible matrix, I is a k x k identity matrix, 0 is an m x k zero matrix, and T is an arbitrary k x m matrix. Show that A has an inverse and that

$$A^{-1} = \begin{vmatrix} B^{-1} & 0 \\ -TB^{-1} & I \end{vmatrix}$$

8. (10%) Construct a general solution of the system Ax = b. A is an m x n matrix with rank m; b is a column vector; both A and b are assumed to take known value; and x is a row vector to be determined.

9. (10%) Show that the set of feasible solutions to the following system forms a convex set.
   Minimize: cx
   Subject to Ax = b, where Ax = b is defined in the above Problem
   $x \geq 0$

10. (10%) Which of the following functions are convex, concave, or neither? Give your answer with details.

a).(2%) $f(x_1,x_2) = e^{-x_1-x_2} + x_1^2 - 2x_1$

b).(4%) $f(x_1,x_2) = Maximum[\, x_1^2+x_2^2,\ 2x_1^2-x_2 \,]$

b).(4%) $f(x_1,x_2) = Minimum[\, x_1^2+x_2^2,\ 2x_1^2-x_2 \,]$