1. (10%)

Consider two urns. The first containing two white and seven black balls, and the second containing five white and six black balls. We flip a *fair* coin and then draw a ball from the first urn or the second urn depending upon whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was tails given that a black ball was selected?

2. (10%)

A rat is trapped in a maze. Initially he has to choose one of two directions. If he goes to the right, then he will wander around in the maze for three minutes and will then return to his initial position. If he goes to the left, then with probability 1/3 he will depart the maze after two minutes of traveling, and with probability 2/3 he will return to his initial position after five minutes of traveling. Assume that the rat is at all times equally likely to go to the left or the right. What is the expected number of minutes that he will be trapped in the maze?

3. (10%)

Suppose that X_1 and X_2 are independent exponential random variables with respective means $1/\lambda_1$ and $1/\lambda_2$; then what is $P\{X_1 < X_2\}$?

NOTE: A continuous random variable X is said to have an exponential distribution with parameter λ, $\lambda > 0$, if its *pdf* is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

4. (10%)

Suppose you are planning to randomly sample 15 items from an infinite population where the mean is 100 and where the standard deviation is 20, normally distributed. Compute the standard error of the mean. Determine the probability that the sample mean will be in the range of 97 to 103.

5. (15%)

Myra Lee has earned a reputation of being a Crackerjack golf instructor. It has been claimed that in only three months of weekly lessons, she can take more than 10 strokes off your score. In order to test this claim, you identified nine players at random who took lessons from Myra. You

have recorded their average 18-hole scores both <u>before and after</u> the three months of lessons as follows:

Before	103	110	85	93	101	96	82	86	89
After	86	92	82	86	90	93	83	80	76

(a) (10%) Does this sample support the "More than 10 stroke" claim at a level of significance of 5%?

(b) (5%) How many strokes reduction can you estimate with 0.90 confidence coefficient?

6. (15%)

Chris Wong is owner of the Mammoth Music House. He suspects that orders arrive at the shop for custom drum sets according to a Poisson distribution. Over the past few weeks at randomly selected hourly periods, Chris has collected the following information:

Orders per hour	0	1	2	3	4	5 or more
Number of observations	30	25	30	24	10	6

(a) (5%) Compute the expected probability of arrivals per hour assuming $\lambda = 2$ arrivals per hour.

(b) (5%) Compute the χ^2 statistic.

(c) (5%) At the 0.05 level of significance, is Chris justified in describing order arrivals according to a Poisson distribution?

Remark: $e^{-2} = 0.13534$, $\chi^2_{5,0.05} = 11.0705$, $\chi^2_{4,0.05} = 9.48773$, $t_{8,0.05} = 1.86$

7. (30%)

A linear regression model $Y_i = 10 + \beta X_i + \varepsilon_i$ is to be estimated by the following data: $(X,Y) = \{(1,9), (3,8), (5,5), (7,2), (9,1)\}$.

(a) (10%) Find the least square estimates of β and $Var(\varepsilon_i)$.

(b) (10%) Find the 95% confidence interval for $E[Y|X=1]$, $E[Y|X=3]$, $E[Y|X=5]$, $E[Y|X=7]$, and $E[Y|X=9]$.

(c) (10%) Estimate the area of the triangle defined by the regression line, the x-axis, and the y-axis. Find approximately the 95% confidence interval for the estimate of area.