1. (10%) Evaluate the following iterated integrals:

a) $\int_{1}^{3}\int_{-x}^{3x}\int_{ye^{x}}^{ye^{2x}} dz\,dy\,dx$

b) $\int_{0}^{4}\int_{x}^{2\sqrt{x}} y^{2}\,dy\,dx$

2. (10%) Find the local extrema (if they exist) of the following functions. Identify the local extrema whenever possible.

a) $f(x,y) = x^2 + y^2 - 2x + 3y + 7$

b) $f(x,y) = xy + (x+y)(10 - x - y)$

3. (10%) Given that $z = f(x,y)$ is a function implicit in $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial x \partial y}$, and $\frac{\partial^2 z}{\partial y^2}$.

4. (10%) Determine the value of x for which the following series are convergent or divergent:

a) $\sum \frac{\sin x}{n}$

b) $\sum (-1)^{n-1} n x^{2n+1}$

5. (10%) Show that if $f:[a,b] \to R$ is continuous, then there exists a positive number M such that $x \in [a,b] \Rightarrow |f(x)| \le M$.

(背面仍有題目,請繼續作答)

6. (10%) Determine if W is a subspace of V based on the given conditions. Give your answers with details.

(a) (2%) Let $V = \mathbb{R}^3$. $W = \{(a,b,0): a,b \in \mathbb{R}\}$, i.e. W is the xy plane consisting of those vectors whose third component is 0;

(b) (4%) Let $V = \mathbb{R}^3$. $W = \{(a,b,c): a^2+b^2+c^2 \le 1\}$, i.e. W consists of those vectors whose length does not exceed 1;

(c) (4%) Let V be the vector space of all square $n \times n$ matrices over a field K. W consists of all matrices which commute with a given matrix T; that is, $W = \{A \in V: AT = TA\}$.

7. (10%) Find the dimension and a basis of the solution space W of the system

$$\begin{aligned} x+2y+2z-s+3t&=0 \\ x+2y+3z+s+t&=0 \\ 3x+6y+8z+s+5t&=0 \end{aligned}$$

8. (10%) Let T be the linear operator on $\mathbb{R}^2$ defined by T(3, 1) = (2, -4) and T(1, 1) = (0, 2). Find T(a, b).

9. (10%) Consider two bases of $\mathbb{R}^2$: $\{e_1=(1,0), e_2=(0,1)\}$ and $\{f_1=(1,3), f_2=(2,5)\}$.

(a) (3%) Find the transition matrix P from $\{e_i\}$ to $\{f_i\}$.

(b) (3%) Find the transition matrix Q from $\{f_i\}$ to $\{e_i\}$.

(c) (4%) Show that $[v]_f = p^{-1}[v]_e$ for any vector $v \in \mathbb{R}^2$, where $[v]_f$ and $[v]_e$ are the coordinate vectors of v relative to $\{f_i\}$ and $\{e_i\}$, respectively.

10. (10%) Find all eigenvalues and a basis of each eigenspace of the operator $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z) = (2x+y,\ y-z,\ 2y+4z)$.