1. (10%) Let $y = f(X) = X^3$, $X = 5$, $\Delta X = 0.1$. Compute and Compare df and Δf.

2. (15%) Maximize the function $X^2 Y$ for the points (X, Y) on the circle $X^2 + Y^2 = 1$

3. (10%) Obtain a recursion formula for $\int \tan^n \theta \, d\theta$

4. (15%) Let M_2 be a number such that $\left| f''(x) \right| \leq M_2$ for all x in [a,b]. Then the error involved in using the trapezoidal formula to approximate $\int_a^b f(x) \, dx$ has absolute value at most $h^2 M_2 (b-a)/12$, where $h = (b-a)/n$, please prove such theorem.

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5 (10%) Give a definition for the following terms: a). Linear Independent, b). Concave function, c). Quadratic Form of a function, and d). Principal Minor of a matrix.

6. (10%) Let B be an invertible matrix with nonnegative entries. Show that every row of B^{-1} has at least one positive entry.

7. (10%) Show that a hyperplane $H = \{ x : px = k \}$ and a halfspace $H^+ = \{ x : px \geq k \}$ are convex set.

8 (10%) Show that function $f(x)$ is convex IF AND ONLY IF its epigraph = $\{ (x, y) : x \in E^n, y \in E^1, y \geq f(x) \}$ is a convex set.

9 (10%) Show that C is a convex cone IF AND ONLY IF x and $y \in C$ imply that $\lambda x + \mu y \in C$, for all $\lambda \geq 0$ and $\mu \geq 0$.