1.(25%)

(1)若相鄰到達者的間隔時間(Inter-arrival time)是指數分配，其期望值是 30 分鐘，問隨機變數 Y 是每小時到達的人數，則 Y 的機率函數是什麼？(5%)

(2)令 X_1，X_2，....，X_{100}是抽自超幾何分配(Hypergeometric distribution)之隨機樣本，其平均值 E(X)=2.8，變異數 V(X)=0.56，試求 $Y=\frac{\sum_{i=1}^{100}X_i}{100}$ 之近似分配是什麼？試加以說明理由。(5%)

(3)設隨機變數 X 服從二項分配 $b(x;n,p)=\binom{n}{x}p^x(1-p)^{n-x}, x=0,1,\ldots\ldots,n.$

已知 n=40，p=0.05，問 $\lim_{n\to\infty}\binom{n}{x}p^x(1-p)^{n-x}$ 之近似分配是什麼？試加以說明（或推導）之。(5%)

(4)連續投擲一公正之六面骰子為實驗，令 X 表示其首次出現么點面時的總試行次數，試求 $p(2\le x\le 3)=$？(5%)

(5)若隨機樣本 X_1，X_2，....，X_{10}是抽自下列 poisson 分配 $p(x;\theta)=\frac{e^{-\theta}\cdot\theta^x}{x!}, x=0,1,2,\ldots\ldots$ 試求條件機率：

$p(X_1+X_2=3\mid\sum_{i=1}^{10}X_i=5)=$？(5%)

(背面仍有題目,請繼續作答)

2.(25%)某連鎖店經理爲了瞭解廣告的促銷效果，隨機抽取 20 家分店比較廣告前的銷售量 Xi 與廣告後的銷售量 Yi ， i=1,2,….,20 ，之情況，得下列的數據：

廣告前的銷售量 Xi	廣告後的銷售量 Yi
樣本平均數 $\bar{x}=1000$	樣本平均數 $\bar{y}=1200$
樣本變異數 $S_x^2=\frac{1}{n-1}\sum(x_i-\bar{x})^2=1600$	樣本變異數 $S_y^2=\frac{1}{n-1}\sum(y_i-\bar{y})^2=2500$

又得知廣告前後銷售量的樣本相關係數 $\gamma=0.9$ 。

假設回歸模式：$Y_i=\beta_0+\beta_1x_i+\varepsilon_i$，其中 $\varepsilon_i\sim NID(0,\sigma^2)$， i=1,2,….,20.

試作下列問題：

(1)求 β_1 的最小平方估計值(5%)

(2)估計 σ^2 的值(5%)

(3)試用顯著水準 $\alpha=0.05$，檢定廣告前的平均銷售量 μ_x 與廣告後的平均銷售量 μ_y 是否相等？(假設 X 、 Y 的母體均滿足變異數相等常態分配)(5%)

(4)在 $\alpha=0.05$ 下，檢定 $H_0:\beta_1=1$ 對 $H_a:\beta_1\neq 1$ (5%)

(5)已知廣告前的銷售量是 980 ，求其廣告後的平均銷售量是多少？(5%)

已知 $t_{16,0.025}=2.120$ ， $t_{17,0.025}=2.110$ ， $t_{18,0.025}=2.101$

$t_{38,0.025}=2.022$。

3. (6%) Consider a confidence interval, with confidence coefficient $1-\alpha$, for the mean of a normal distribution with known variance σ^2, based on a random sample of n observations. Explain how the width of the interval changes

(i) (2%) as n is increased, keeping σ^2 and α fixed;

(ii) (2%) as σ^2 is increased, keeping n and α fixed;

(iii) (2%) as α is decreased, keeping n and σ^2 fixed.

4. (14%)

(i) (4%) Describe the method of maximum likelihood.

(ii) (10%) If X denotes the number of the trial on which the first defective is found in a series of independent quality-control tests, find the maximum likelihood estimator of p, the true probability of observing a defective.

5. (15%) A discrete random variable Y takes the values -1, 0 and 1 with probabilities $\frac{1}{2}\theta$, $1-\theta$ and $\frac{1}{2}\theta$ ($\theta \in (0,1)$), respectively. Let Y_1 and Y_2 be two independent random variables, each with the same distribution as Y.

(i) (5%) List the possible values of $\{Y_1, Y_2\}$ that may arise and calculate the probability of each. Verify your answers.

(ii) (5%) By calculating the value of $(Y_2 - Y_1)^2$ for each possible pair $\{Y_1, Y_2\}$, determine the probability distribution of $(Y_2 - Y_1)^2$.

(iii) (5%) Let $X = \frac{1}{2}(Y_2 - Y_1)^2$, find the variance of X, $V(X)$, as a function of θ.

6. (15%) Suppose that $\hat{\theta}_1$ and $\hat{\theta}_2$ are each unbiased estimators of θ, with variances $V(\hat{\theta}_1) = \sigma_1^2$ and $V(\hat{\theta}_2) = \sigma_2^2$, respectively. A new unbiased estimator for θ can be formed by

$$\hat{\theta}_3 = a\hat{\theta}_1 + (1-a)\hat{\theta}_2,$$

where $0 \le a \le 1$. If $\hat{\theta}_1$ and $\hat{\theta}_2$ are independent, how should a be chosen so as to minimize $V(\hat{\theta}_3)$?