IMS - M.S. Entrance Exam (1998)

Calculus

1. (10%) Find all values of x for which the series $\sum \frac{(x-2)^n}{n}$ converges. Distinguish between absolute and conditionally convergence.

2. (10%) Explain why L'Hôpital's rule does not apply to $\lim_{x\to\infty} \frac{x+\cos x}{x}$, evaluate the limit.

3. (10%) Find the volume generated by the region below the curve $y = \sin x$, $0 \le x \le \pi/2$, is rotated about the y axis, or conclude that it does not exist.

4. (10%) Sketch the graph of $y = x^4 - 4x^3 + 15$, including all turning points and points of inflection.

5. (10%) What is the maximum value of $f(x) = 1 - x^{3/2}$ in the domain [0, 1]? What is the minimum value? Does $f'(x) = 0$ at the points where these extreme values occur?

(背面仍有題目,請繼續作答)

6. **(10%)** Prove that if A is symmetric and invertible, then A^{-1} is also symmetric.

7. **(10%)** The function g: $P_2 \to P_3$ is defined as follows:

$$g(a_2x^2 + a_1x + a_0) = (a_2 - a_1)x^3 - a_1x + 2a_0$$

a) Prove that g is linear.

b)Find the kernel and range of g and give bases for these subspaces.

8. **(10%)** Let $T: U \to V$ be a linear transformation. Let T be defined relative to bases $\{u_1, u_2\}$ and $\{v_1, v_2, v_3\}$ of U and V as follows: $T(u_1) = v_1 + 5v_2 - 2v_3$, $T(u_2) = 3v_1 - v_2 + 2v_3$.

a)Find the matrix of T with respect to these bases.

b)Use this matrix to find the image of vector $u = 2u_1 - 3u_2$.

9. **(10%)** Diagonalize the symmetric matrix $\begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$. Give the similarity transformation.

10. **(10%)** Determine a)the point-normal form and b)the general form of the equation of the plane through the points (1, -3, 2) and having normal (1, -1, 3)