

1. Show that $f(x) = |x^2 - 4|$ is continuous but not differentiable at $x = 2$. (15%)

2. Find the derivative of $f(x)$:

$$f(x) = \frac{\ln(5x+1)^2}{x^2+1} + \int_x^\infty 2(y-x)e^{-x^2y} dy$$

You do not need to integrate out the integral. (15%)

3. Suppose $f(x)$ is differentiable. Find its derivative from the following equation. (10%)

$$x^3 - x = [f(x)]^3 - [f(x)]^2 + 24$$

4. Calculate $\sum_{n=1}^{\infty} n(0.6)^n$. (10%)

5. Show that $\lim_{x \rightarrow x_0} \sin(x+c) = \sin(x_0+c)$ (10%)

6. Show that $\lim_{x \rightarrow 0^+} x^x = 1$ (10%)

7. Find the sum of $\sum_{n=0}^{\infty} (n+1)x^n$, ($|x| < 1$), (10%)

8. Evaluate $\int \frac{e^x dx}{\sqrt{2+e^{2x}}}$ (10%)

9. Suppose $f: [a, b] \rightarrow \mathbb{R}$ is continuous. Show that $m(b-a) \leq \int_a^b f \leq M(b-a)$, where M and m are the maximum and minimum of f on $[a, b]$, respectively. (10%)