

1. (10%) Find the tangent line to the curve of intersection of the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ and the sphere $x^2 + y^2 + z^2 = 6$ at the point $(1, 1, 2)$.
2. (10%) Find the following limits if they exist.
 - (a) $\lim_{x \rightarrow +\infty} x(k^{1/x} - 1)$ where k is a positive constant
 - (b) $\lim_{x \rightarrow 0} \frac{x^2 \sin(1/x)}{\sin x}$
3. (10%) Suppose that the point $(2, 1)$ is on the curve $x^2y + ay^2 = b$ and the tangent line at $(2, 1)$ has the equation $2x + 3y = 5$. Find the values of a and b .
4. (10%) Sketch the graph of $f(x) = (x \ln x)/(x^2 + 1)$ and find the relative extrema of $f(x)$ on the interval $(0, \infty)$.
5. (10%) Find the third-order Taylor polynomial of $\sin(\tan^{-1} x)$ at $x_0 = 1$.
6. (10%) Find $\int \frac{dx}{1 - \sin x + \cos x}$.
7. (10%) The semi-circular region $R = \{(x, y) : x^2 + y^2 = 16, y \geq 0\}$ is covered by a thin titanium alloy whose density at (x, y) is $\delta(x, y) = y$. Find the center of mass of the material.
8. (10%) Use a power series based at $a = 0$ to find the solution of $y' = y$, satisfying the initial condition $y = 1$ when $x = 0$.
9. (10%) The flight of a missile is given by the parametric equations $x = 2t^2, y = t^3, t \geq 0$, where t is time. When $t = 1$, the ground control spies a target at the point $(5, 3)$ and the missile flies off on a tangent. Will it reach the target?
10. (10%) In a semi-conductor factory, a tank contains 100 liters of contaminated water holding 10 kilograms of dissolved toxic chemicals. Water containing 0.02kg/liter of chemical enters the tank at the rate of 2 liters per minute and the contaminated water (kept uniformly mixed by stirring) flows out at the same rate. The contaminated water can be discharged to the recycling system only when the amount of chemical in the tank is less than 3 kg. Determine the time to empty the tank.