1. (12%) A computer store has received two shipments of monitors. The first shipment contains 100 monitors, 4% of which are defective. The second shipment contains 50 monitors, 6% of which are defective. Suppose that Sam picks off a monitor at random of the shelf and purchases it, and he later discovers that the monitor he purchased is defective. Is the defective monitor more likely to come from the first shipment or from the second shipment? (A detailed analysis is required.)

2. (12%) There are three sealed and opaque boxes, each containing a ball. These three balls are identical except that two balls are red and one ball is green. If you pick the box that contains a green ball, you will receive $10,000 in reward. Suppose you picked one of the boxes at random, and then one of the other boxes was opened which contains a red ball. You are now offered an option to exchange the boxes (between the sealed box you picked and the remaining sealed box). Should you change your pick? (A detailed analysis is required.)

3. (a) (3%) Suppose X is a random variable with expected value μ and variance σ^2, and Y is a linear function of X, say $Y = aX + b$, where a and b are constants. What is the covariance of Y and X?
(b) (3%) Suppose $\bar{X} = (X_1 + \cdots + X_n)/n$ where $X_1, \ldots, X_n$ are independently and identically distributed random variables with expected value μ and variance σ^2. What is the covariance of $\bar{X}$ and $(\bar{X} - X_i)$, $i = 1, \ldots, n$?

4. Consider the moment generating functions of random variables.
(a) (3%) Will the moment generating functions of two random variables with different distribution functions have the same form? Please explain.
(b) (3%) Suppose that $(pe^t + 1 - p)^n$ and $(pe^t + 1 - p)^m$ are the moment generating functions of random variables X and Y, respectively. Which distribution does the moment generating function $(pe^t + 1 - p)^{n+m}$ refer to, in terms of X and Y?

5. Multiple-choice questions; no credit will be given if the answer is incorrect.
(a) (3%) Consider a discrete random variable X that takes on one of the integral values $x_1, x_2, \ldots$, where $x_1 \le x_2 \le \cdots$. Which of the following statements regarding X would be correct?
   (1) An outcome x_i can be associated with different probabilities.
   (2) $P(X = x_i)$ for a given i is called the probability distribution function of the random variable X.
   (3) $P(X = x_i) = P(X \le x_i) - P(X \le (x_i - 1))$.
   (4) In the graph of the probability distribution of X, the Y-axis values of all x_i can be close to 1.
(b) (3%) Consider the pdf, $f(x)$, and cdf, $F(x)$, of a continuous random variable X. Which of the following statements would be correct?
   (1) $f(x)$ could be increasing.
   (2) If X obeys a uniform distribution over a range from p to q, then $f(x)$ is always $1/(q-p)$.
   (3) The Y-axis of $F(x)$ of X for some value t indicates $P(X = x)$.
   (4) From the graph of $F(x)$, it is impossible to identify which values that X takes on occur more frequently.

6. (8%) Suppose that X and Y are independent random variables and that X is uniformly distributed on (0,2) and Y is uniformly distributed on (1,3). What is the probability density of $X + Y$? (A detailed analysis is required.)

7. (10%) In the large sample case for hypothesis testing on a population mean μ, if the null hypothesis is H_0: $\mu \le \mu_0$ for some specific constant μ_0, explain why it is appropriate to reject the null hypothesis when the test statistic $z > z_\alpha$ for confidence level $1-\alpha$.

8. (10%) In the goodness of fit test, let f_j and e_j be the observed frequency and expected frequency of category j, respectively. Then the test statistic is $\chi^2 = \sum_j \frac{(f_j - e_j)^2}{e_j}$. Argue whether the conditions $e_j \ge 5$ for every j are necessary.

9. (12%) The assumptions about the error term ε in a simple linear regression model $y = \beta_0+\beta_1 x+\varepsilon$ are: (i) $E(\varepsilon) = 0$; (ii) the variance of ε is the same for all values of x; (iii) the values of ε are independent; and (iv) ε has a normal probability distribution. A residual plot against variable x is a scatter diagram in which the values of x are represented by the horizontal axis and the corresponding residual values are represented by the vertical axis. Discuss how many assumptions given above can be identified from the residual plot against variable x, and justify your answer.

10. (10%) The exponential smoothing model for forecasting is $F_{t+1} = F_t + \alpha(Y_t - F_t)$, where F_t is the forecast of period t, Y_t is the actual value of period t, and α is a smoothing constant between 0 and 1. Why can this model exponentially smooth forecasts? (That is, interpret "exponential" and "smoothing" individually.)

11. (8%) Suppose that a population has an exponential distribution with mean μ. Let $\{x_1, x_2, \ldots, x_n\}$ be a set of independent observations from the population. Argue whether $\bar{x} = \sum_{i=1}^{n} x_i / n$ is an unbiased and consistent estimator of μ, and justify your answer.