1. (10%) Using the basic definition of derivative, show that in the domain where functions f and g are both differentiable, $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$.

2. (10%) Rigorously show that the mean value theorem applies to $f(x)=\sqrt{x}$ in the interval [0, 4]. Find the mid point c that satisfies the theorem.

3. (10%) In an integrated circuit fabrication process, a spherical drop of etching chemical loses moisture by evaporation at a rate proportional to its surface area. What can you say about its radius?

4. (10%) A manufacturer receives an order for oil cans that are to have a capacity of k cubic centimeters. Each can is made from a rectangular sheet of metal by rolling the sheet into a cylinder; the lids are stamped out from another rectangular sheet. What are the most economical proportions of the can that minimizes the cost of metal sheet (Proportional to its area)?

5. (10%) Using the chain rule only, find $\frac{d}{dx}\int_0^{\sin x}(1-t^2)dt$.

(背面仍有題目,請繼續作答)

6. Solve by Newton's method: $x^2 - 7 = 0$. (10%)

7. Compute $\lim_{h \to 0^+} (1+ah)^{b/h} \quad (a > 0, b > 0)$ (10%)

8. Compute $\sum_{n=3}^{\infty} \left(\frac{1}{(n+1)(n-2)} + \frac{2^n - 1}{3^n} \right)$ (10%)

9. Solve $y'' + 5y' - 6y = 0$ and $y(0) = 1, y'(0) = 2$ (10%)

10. Compute the volume of the given set using integration: Bounded above and below by the planes $x - 2y + 3z = 4$ and $x - 2y + 3z = 2$, and laterally by the cylinder $x^2 + (y-1)^2 = 1$. (10%)