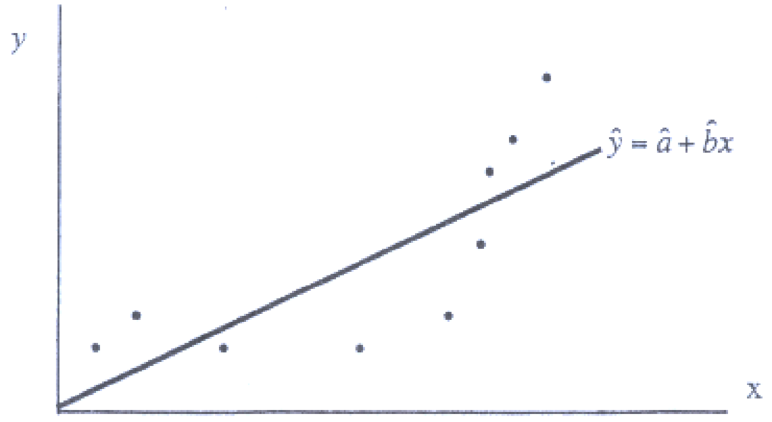
1. (10%) For the random variable $f_X(x) = \dfrac{1}{\theta}$, $0<x \le \theta$, and $0<\theta<\infty$, derive the Maximum Likelihood Estimator $\hat{\theta}$ of θ. Use the definitions of unbiased estimator and consistent estimator to show that $\hat{\theta}$ is unbiased or biased, consistent or inconsistent.

2. (10%) A total of n independent samples are taken from an exponentially distributed random variable with a given parameter $\lambda=2$. Propose an appropriate statistical test to determine whether $\hat{\lambda} = \lambda$ or not, where $\hat{\lambda}$ is the estimator from the samples. Is this an exact or approximate test? Also propose an appropriate statistical test to test the hypothesis that the samples come from the designated exponential distribution. What are the specific purposes of this test?

3. (10%) Two different catalysts are being analyzed to determine how they affect the mean yield of a wafer etching process. It is desirable to detect which catalyst to use, to increase the yield. Two statistical tests are considered for this experiment: the two-sample t-test (independent t-test) and the paired t-test; and the number of test samples is n for each catalyst. Describe the differences of the two selected tests in terms of: the experimental order, the statistical model, and the test hypothesis. When is it appropriate to use the paired t-test?

4. (10%) Describe the assumptions used in the development of the Analysis of Variance model. How to detect the departure from these assumptions in the model, and how to rectify the problems encountered during the process?

5. (10%) Nine data in the figure below have been fitted into a simple linear regression model $y = a + b\,x$. Describe the model inadequacies and the methods to detect it. What have you gained or lost when a higher order (e.g., 2nd or 3rd order) model is used in the model fitting process? Explain in details.



(背面仍有題目,請繼續作答)

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6. (10%) For any two events E and F, show that $P\{E \cap F\} \geq P\{E\} + P\{F\} - 1$ and interpret this inequality.

7. Let U_1, U_2, U_3, and U_4 be independent uniform random variables defined on (1,2), and let $X = \text{Max}\{U_1, U_2, U_3, U_4\}$.
   (a) (4%) Derive the probability density function of X.
   (b) (6%) Calculate the mean and median of X.

8. Let X and Y be two random variables.
   (a) (3%) What is the meaning of the independence between X and Y?
   (b) (6%) Suppose that both X and Y are discrete. Explain why X and Y are independent when condition $P\{X = x, Y = y\} = P\{X = x\}P\{Y = y\}$ holds for all possible x and y.
   (c) (6%) Suppose that both X and Y are continuous. Explain why X and Y are independent when condition $P\{X \leq x, Y \leq y\} = P\{X \leq x\}P\{Y \leq y\}$ holds for all possible x and y.

9. Suppose that a system with three components 1, 2, and 3, as shown in the following figure, is on in the morning and off at night every day until the system fails. A technician inspects the system every night to replace failure components by new ones. The system fails whenever there does not exist any path from A to B. The failure probability of each component is 0.1 in a day. Component 1 works independently to components 2 and 3. The failure probability of component 2 given that component 3 fails is 0.4, and vice versa.
   (a) (3%) Define the sample space of this experiment.
   (b) (2%) Define a random variable for the system failure probability.
   (c) (5%) Calculate the system failure probability.
   (d) (5%) What is the expected lifetime of this system?