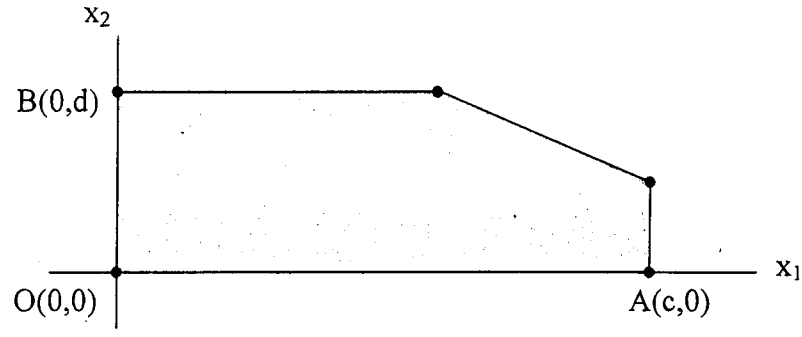
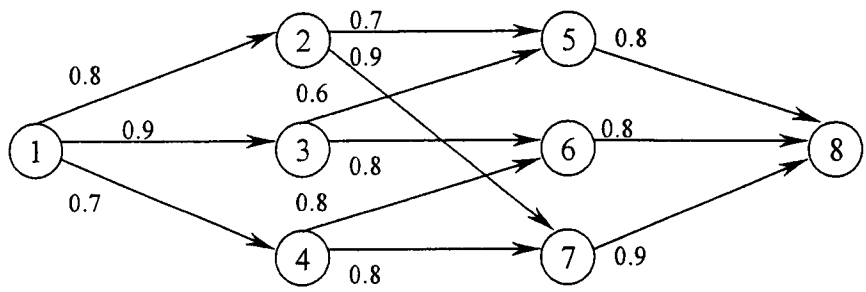
1. (10%) Suppose that a company attempts to solve a problem for maximizing its net profit $Z = ax_1+bx_2$. Let the feasible region of the linear programming model corresponding to this problem be the shaded area shown in the figure given below, where A and B represent two corner-point feasible (CPF) solutions (c,0) and (0,d), respectively. By applying the simplex method starting at O(0,0) to solve this model, when $b > a > 0$ and $c > d > 0$, the next CPF solution will be point A or point B, and justify your answer.



2. (10%) A basic feasible (BF) solution of a linear programming model can be degenerate. Draw a figure in a 2-dimensional space to show that one of the BF solutions is degenerate, and justify your answer.

3. (10%) In the transportation simplex method, we need the values of the u_i for all rows and the v_j for all columns to perform the optimality test. Argue that the choice of the starting basic variable for deriving the values of the u_i and the v_j is arbitrary and will not affect the result of the optimality test.

4. The following figure shows a computer network with 8 stations in a city. A user wants to send a message from station 1 to station 8. The success connection probability between any two stations is shown in the figure given below. Suppose that the user can assign the path for transferring the message.



(a) (6%) Use dynamic programming to find the most reliable path for the user. (Show the process.)
(b) (8%) Select an appropriate network analysis method to solve this problem. (Show the process.)
(c) (6%) For the two methods used in (a) and (b), discuss which one is a better approach for this problem.

(背面仍有題目，請繼續作答)

5 (15%) Parts arrive at a production facility and wait in the buffer area until a total number of k parts have accumulated. Upon the arrival of the k^{th} part, all are instantaneously processed by the machine, and the process repeats. Let $\xi_0, \xi_1, \ldots$ denote the arrivals of parts in successive periods, assumed to be independent random variables whose distribution is given by $\Pr\{\xi_k = 0\} = \alpha$, and $\Pr\{\xi_k = 1\} = 1-\alpha$, where $0 < \alpha < 1$. Let X_n denote the number of parts in the system at time n.

(1) State assumptions required so that the production problem can be modeled as a Markov chain

(2) Show that $\{X_n: n = 0, 1, 2, \ldots\}$ is a Markov Chain

(3) Derive the transition probability matrix

6 (15%) Machines in a factory break down at an exponential rate of 4 per hour. A single repairman can fix machines at an exponential rate of 5 per hour. The operating cost (including the wage) is \$80 per hour. If a semi-automatic repair station is installed, the failed machine can be fixed at an exponential rate of 6 per hour. However, the hourly operating cost will be increased to \$160 (including the wage of the repairman and operating cost of the repair station). The cost incurred in lost production when machines are failed is \$60 per hour per machine.

(1). State the assumptions regarding the number of machines in the system, when M/M/1 model is used to solve this production problem.

(2). Construct the rate diagram and determine the repairman's utilization for the two cases.

(3). Compute the total hourly cost for the two alternatives. Do you recommend the factory to use the semi-automatic repair station?

7 (10%) Give a definition for each of the following terms: Inventory carrying cost, Shortage cost (two types), Reorder point.

8 (10%) Give a definition for each of the following terms: Linear congruential generator, Time statistics. Describe two methods to generate random variates from a probability distribution.