本試題是否可以使用計算機： ☑可使用 ， □不可使用 （請命題老師勾選）

1. (38%) Consider the problem of preparing batches of a recipe in a food processing plant. Each batch is a mixture of 6 powdered ingredients denoted by I_1 to I_6 according to the following recipe

Ingredient I_i	I_1	I_2	I_3	I_4	I_5	I_6
kg needed per batch	130	146	101	290	20	21
Flow rate g_i (kg/second)	1	1.2	1.3	1.1	0.8	0.9

There are 3 weighing machines WM1, WM2, WM3 working simultaneously and independently in the plant. The ingredients are held in sizeable hoppers attached to the weighing machines. Some ingredients flow "faster" than others from the hoppers. g_i given in the above table is the number of kg of ingredients I_i that flows from the hopper onto the scales of the weighing machine per second. There are 3 hoppers on WM1, 2 on WM2, and 4 on WM3. The ingredients held in these hoppers are indicated with a * in the following table.

	I_1	I_2	I_3	I_4	I_5	I_6
WM1	*		*	*		
WM2		*		*		
WM3	*	*			*	*

An ingredient can be weighed on a weighing machine only if one of its hoppers contains that ingredient. The required quantity of each ingredient in a batch can be split among all weighing machines on which it can be weighed. (For example, the 130 kg of I_1 needed per batch can be split among WM1 and WM3.) Once the quantity of each ingredient to be weighed on each weighing machine is determined, each weighing machine begins its work independently. On each weighing machine, the ingredients are emptied from one hopper at a time into its scales and weighed, in the order from left to right. (For instance, I_1 will be weighed first, I_3 next, and I_4 last on WM1.) When every weighing machine has completed weighing the quantities of all ingredients allocated to it, the contents of the scales on all of them are conveyed to a bin where they are blended into the batch. Then the scales are dusted off and preparations begin for the next batch.

Formulate the problem of determining the quantities of each ingredient to be weighed on each of the weighing machines, to minimize the time needed to prepare a batch, as an LP. (No credit will be given unless the decision variables, the constraints, and the objective function(s) are clearly defined, and each constraint is justified.)

2. (12%) State whether the following statements are true or false, and if false, explain why.
   a. (4%) The minimum ratio in a pivot step of the primal simplex algorithm is always the value of the entering variable in the next BFS.
   b. (4%) In the original tableau for an LP in standard form, if there is a column with all negative entries, and a negative cost coefficient, the objective function is unbounded below in this LP.
   c. (4%) In a degenerate pivot step of the primal simplex algorithm, the values of the basic variables, the objective value, and the relative cost coefficients do not change.

（背面仍有題目,請繼續作答）

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3. (10%) Give a definition for the following terms: Stochastic process, Little's formula, Birth and death process.

4. (10%) State the conditions that a Markov chain must satisfy so that steady-state probabilities exist.

5. (15%) The replenishment of stock for a digital camera to satisfy demand takes place at the end of periods (days) labeled n=0, 1, 2, . . .; and the daily demand of camera is an independent random variable with P(demand of 0)=0.3, P(demand of 1)=0.5, and P(demand of 2)=0.2. The inventory policy is (s=0, S=3), i.e., if the end of period stock quantity is no greater than 0, then an amount sufficient to increase the quantity of stock on hand up to the level of 3 is immediately procured. An unfilled demand is immediately satisfied upon restocking. If, however, the available stock is in excess of 0, then no replenishment of stock is undertaken. Let X_n denote the quantity on hand at the end of period n just prior to restocking. Formulate this inventory problem as a Markov Chain.
   a. Specify the state space and specify the transitional probability matrix P.
   b. Determine the long run fraction of periods in which demand is not met.
   c. Determine the long run average inventory level.

6. (15%) Consider a network of three workstations. Parts arrive at stations 1, 2, 3 in accordance with Poisson processes having respective rates 5, 10, 15. The processing times at the three stations are exponential with respective rates 10, 50, 100. A part completing processing at station 1 is equally likely to either (1) go to station 2, (2) go to station 3, or (3) leave the system. A part departing service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to either go to station 2 or leave the system.
   a. What type of network of queue does this problem fit?
   b. Determine the actual total arrival rate at each service station.
   c. Determine the utilization level of each server.
   d. Determine the limiting probabilities. If the probabilities do not exist, state the reason.
   e. Determine the expected time in system for each part and the expected number of parts in the system.