系所：工業與資訊管理學系甲組 科目：作業研究

本試題是否可以使用計算機： □可使用 ， ☒不可使用 （請命題老師勾選） 考試日期：0302，節次：2

1. Name five important persons and briefly describe their contributions in the development of mathematical programming. (15%)

2. For the following linear program with bounded variables, formulate its dual.

   max. $\boldsymbol{c}^t\boldsymbol{x}$

   s.t. $\boldsymbol{A}\boldsymbol{x}\leq\boldsymbol{b}$

   $\boldsymbol{l}\leq\boldsymbol{x}\leq\boldsymbol{u}$

   where $\boldsymbol{A}$ is a constant matrix, $\boldsymbol{c}$, $\boldsymbol{b}$, $\boldsymbol{l}$, and $\boldsymbol{u}$ are constant column vectors, and $\boldsymbol{x}$ is a variable column vector. (15%)

3. For the following mathematical program:

   max. $x_1 + 2x_2$

   s.t. $x_1 + x_2 \leq 4$

   $3x_1 + 2x_2 - |2x_1 - x_2| \leq 6$

   $x_1, x_2 \geq 0$

   (a) Transform it into a conventional linear program which can be solved by LP computer packages. (10%)

   (b) Use any method to solve. (10%)

4. (30%) Consider a workshop consisting of two machines. The potential jobs arrive in accordance with a Poisson process at rate 2, and that the service times for the two machines are independent and have respective exponential rates of 2 and 3. Suppose that an entering job first will processed on machine 1. When its work is completed on machine 1, it will go either machine 2 if that machine is idle or else wait in machine 1 until machine 2 becomes available. Suppose that a potential job will enter this workshop as long as machine 1 is idle.

(a) What proportion of potential jobs enters the workshop? (10%)

(b) What is the mean number of jobs in the workshop? (10%)

(c) What is the average amount of time that an entering job spends in the workshop? (10%)

5. (10%) Trial are performed in sequence. If the last two trials were successes, then the next trial is a success with probability 0.8; If the last two trials were fail, then the next trial is a success with probability 0.3; Otherwise the next trial is a success with probability 0.5. In the long run, what proportion of trials are success?

6. (10%) Let p_{ij}'s be the one step transition probabilities of a Markov chain with $M+1$ states. If this Markov chain is irreducible, aperiodic, and

$$\sum_{i=0}^{M} p_{ij} = 1, \text{ for all } j.$$

Show that the limiting probabilities

$$\pi_j = \frac{1}{M+1}, \text{ for } j = 0, 1, \ldots, M.$$