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1. (38%) eCatalog is a catalog & mail order company that needs a combination of storage and office space for its products. Tom is in charge of making the decisions on the amount of storage space (x_1) and office space (x_2) to rent for the year. He has to solve the following linear program to determine the optimum values for x_1 and x_2:

$$\begin{aligned} \text{Minimize} \quad & W = 100x_1 + 300x_2 \\ \text{s.t} \quad & x_1 + x_2 \geq 10 \\ & x_1 + 2x_2 \geq 15 \\ & 6x_1 + x_2 \geq 18 \\ & x_1 - x_2 \leq 10 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

   (1) (7%) Solve the problem geometrically and find the optimal values of x_1 and x_2.

   (2) (7%) Construct the dual problem.

   (3) (6%) Use part (1) to help you determine the optimal basic feasible solution for the dual problem.

   (4) (8%) Construct the final tableaux of the dual problem.

   (5) (10%) Suppose the first constraint in the primal problem becomes

$$x_1 + x_2 \geq 10 + \alpha$$

   What is the allowable range of α for the solution in (4) to stay feasible and/or optimal?

2. (6%) When using the branch-and-bound method to solve a pure integer programming problem, under what conditions do we branch no further on a subproblem?

3. (6%) Briefly describe the procedure of the Big M method.

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4. (20%) Terminology
   (a) Markov chain (5%)
   (b) Poisson process (5%)
   (c) No-memory property (5%)
   (d) (s, S) policy, (R, Q) policy (5%)
5. (10%) Derive the EOQ formula under the deterministic environment.
6. (10%) It is well known that the exponential distribution has the no-memory property. Show that the geometric distribution: $f(x)=p(1-p)^{x-1}$, $x=1, 2,\ldots$, also has the no-memory property.
7. (10%) For an $M/M/\infty$ queueing system with arrival rate λ and identical service rate μ for every server, what is the expected time that a customer spends in the system? What about in the queue? What is the expected number of customers in the system? What about in the queue? Give an example of this type of system in the real world?