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1. (18%) Cheng-Kung company has three branch plants with excess production capacity. Fortunately, the company has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes-large, medium, and small-that yield a net profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rate of the new product. Plant 1, 2, and 3 have 13000, 12000, and 5000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sale forecasts indicate that if available, 900, 1200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wish to know how much of each size should be produced by each plant to maximize profit. Formulate a linear programming model for this problem.

2. (16%) A company has two plants producing a certain product that is to be shipped to three distribution centers. The unit production costs are the same at both plants, and the shipping cost per unit of the product is shown for each combination of plant and distribution center as follow:

		Distribution Center		
		1	2	3
Plant	A	8	7	4
	B	6	8	5

Each plant can produce and ship any amount up to a maximum of 50 units per week.

(a) Assume that any distribution center may receive any quantity between 10 and 30 units per week, however, the total shipped to all three distribution centers equals 60 units per week.

(背面仍有題目,請繼續作答)

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Formulate this problem as a transportation problem by constructing the appropriate cost and requirement table.

(b) Assume that distribution center 1, 2, and 3 must receive exactly 10, 20, and 30 units per week, respectively. For administrative convenience, management has decided that each distribution center will be supplied totally by a single plant. Formulate this problem as an assignment problem by constructing the appropriate cost table.

3. (16%) Consider the follwing nonliner programming problem:

$$\begin{aligned} \text{Maximize} \quad & f(\mathbf{x}) = \frac{x_1}{x_2 + 1} \\ \text{Subject to} \quad & x_1 - x_2 \leq 2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

(a) Obtain the KKT conditions for this problem.

(b) Use the KKT conditions to derive an optimal solution.

4. (20%) Give a brief definition for each of the following terms: Conditional expectation, Random variable, Random walk, and Queuing network.

5. (15%) Two persons A and B start a game with \$3 each as their capital. At the end of every game, the loser pays \$1 to the winner. With every game the probability of A winning is p ($0<p<1$), the probability of B winning is q ($0<q<1$), and the probability of no-win and no-loss is 1-p-q. They quit playing when one of them either loses or wins all. Let X_n denote the capital of player A at stage n, formulate this problem as a Markov Chain model.
a) Describe the state space.
b) Determine the transitional probability matrix P.
c) Specify the classes of the Markov Chain, show your detailed analysis.

6. (15%) A production system processes items according to a Poisson process with rate λ. The storage space is limited to k items and so it shuts down production whenever k items are in stock. Customers arrive at the facility according to a Poisson process with rate μ. Each customer wants one item and will immediately depart either with the item or empty handed if there is no item available.
- Find the proportion of customers that go away empty handed.
- Find the average number of items on the shelf.
- Find the average time that an item is on the shelf.