

系所組別： 交通管理科學系甲、乙、丙、丁組

考試科目： 統計學

考試日期：0220 · 節次：2

※ 考生請注意：本試題 可 不可 使用計算機

1. TRUE or FALSE (25%)

Consider the following normal error simple linear regression model:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i \quad i = 1 \dots n \quad \text{where}$$

 Y_i is the observed response in the i th trial X_i is a known constant, the level of the predictor variable in the i th trial β_0 and β_1 are parameters ε_i are independent $N(0, \sigma^2)$

Let X_h denote the level of independent variable for which we wish to estimate the mean response $E\{Y_h\}$. Let \hat{Y}_h denote the point estimator of $E\{Y_h\}$; $\hat{Y}_{h(new)}$ denote the new observation on response variable at a given level X_h . $\bar{X} = \sum_i X_i / n$; $\bar{Y} = \sum_i Y_i / n$.

Please indicate if the following statement is True or False. (5% for each question)

- (a) The least squares estimators $b_1 = \sum (X_i - \bar{X})(Y_i - \bar{Y}) / \sum (X_i - \bar{X})^2$ and $b_0 = \bar{Y} - b_1 \bar{X}$ of β_1 and β_0 are unbiased and have minimum variance among all unbiased linear estimators.
- (b) The maximum likelihood estimators of β_1 and β_0 are unbiased and have minimum variance among all unbiased linear estimators.
- (c) β_1 indicates the change in the mean of the probability distribution of response variable per unit increase in predictor variable.
- (d) The variability of the sampling distribution of \hat{Y}_h is affected by how far X_h is from \bar{X} .
- (e) The 95 percent prediction interval for the mean of 3 new observations for given X_h is wider than that obtained for a new observation $Y_{h(new)}$.
2. (25%) Let X be a random variable with mean μ , and the variance of X , denoted by $\text{Var}(X)$. Please define a reasonable way of measuring the possible variation of X . **Explain** your answer (in Chinese or English and/or figure)!
3. (25%) Suppose that the number of kilometers that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 kilometers. If a person desires to take a 5000-kilometer trip, what is the probability that he or she will be able to complete the trip without having to replace the car battery?
4. (25%) Determine the maximum likelihood estimator of θ when X_1, \dots, X_n is a sample with density function

$$f(x) = \frac{1}{2} e^{-|x-\theta|} \quad -\infty < x < \infty$$