系所組別： 交通管理科學系乙、丙、丁組

考試科目： 微積分

※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. (10 %) Compute: $\lim_{x\to\infty}(1+\frac{2}{x})^x$.

2. (10 %) Find the derivative of $f(x)=x^{\sin x}$, $x>0$.

3. (a) (5 %) Find $\frac{d}{dx}\int_1^x e^{t^2}\,dt$.

   (b) (5 %) Find $\frac{d}{dx}\int_x^{x^2} e^{t^2}\,dt$.

4. Evaluate the given integrals

   (a) (5 %) $\int(\ln x)^2\,dx$,

   (b) (5 %) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

5. (10 %) Evaluate $\int_{-1}^{1} x^{-1/3}\,dx$.

6. (a) (5 %) Find the Maclaurin series (Taylor series of a function at 0) for e^x and determine the radius of convergence.

   (b) (5 %) Find the sum of the infinite series $\sum_{k=0}^{\infty}\frac{(\ln 2)^k}{k!}$.

7. (10 %) Determine and classify the critical points of the function

$$f(x,y)=4xy-x^4-2y^2.$$

8. (a) (5 %) Evaluate the iterated integral, $\int_0^1\int_0^x \sqrt{1+x^2}\,dydx$.

   (b) (5 %) Evaluate the iterated integral $\int_0^1\int_y^1 e^{x^2}\,dx\,dy$ by first changing the order of integration.

9. (10 %) Find the absolute maximum and minimum values of the function $f(x,y)=x^3-6x^2-y^2+6$ on the region described by $6x^2+y^2\le 6$.

10. (10 %) Evaluate the integral $\int\int_R y+3x\,dA$, where R is the region bounded by $y=3-3x$, $y=1-3x$, $y=x-3$ and $y=x-1$, by making an appropriate change of variables.