

系所組別：交通管理科學系乙、丙組

考試科目：微積分

考試日期：0223，節次：2

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10 points) Evaluate $\lim_{n \rightarrow \infty} (x - \ln x)$
2. (10 points) Evaluate $\lim_{n \rightarrow 0} \frac{x^2 - \tan x}{1 - \cos x}$
3. (10 points) Evaluate $\int_0^1 x^2 \sin^{-1} x \, dx$
4. (10 points) Evaluate $\int_{-1}^1 \sqrt{\frac{1+x}{1-x}} \, dx$
5. (10 points) Find $\frac{dy}{dx}$ for $x = 0$ if $y = \ln u + \tan^{-1} \sqrt{u}$ and $u = e^{-x} \cos 2x$
6. (10 points) If $f(x) = e^{-x^2/2}$, find the local extreme of f , discuss concavity, find the points of inflection and find asymptotes if any.
7. (10 points) Suppose that a business has profit function $p(x, y, z) = 3xz + 6y$ and manufacturing constraint $x^2 + 2y^2 + z^2 \leq 6$. Maximize the profits.
8. (10 points) Given $f(x) = \sin^2 x$, find $f^{(2k)}(0), f^{(2k+1)}(0)$. (Hint: $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$)
9. (10 points) If $z = f(x, y)$ satisfies $x^2 y + z \cos \pi y - xz^3 = 0$, find gradient $\nabla f(x, y)$ at $(2, 0.5)$.
10. (10 points) Find the area of the surface generated by rotating the curve $r = 2 \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$ about the y -axis.