

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

計算題：每題 10 分，合計 100 分

1. Evaluate  $\lim_{x \rightarrow 3} \frac{x}{x-3} \cdot \int_3^x \frac{\sin t}{t} dt$

2. Find the values of the constants  $a$  and  $b$  such that  $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b}-1}{x} = \frac{5}{3}$ .

3. For what values of the constants  $a$  and  $b$  is  $(1,3)$  a point of inflection of the curve  $y = ax^3 + bx^2$ .

4. Evaluate  $\int \frac{dx}{\sqrt{1-e^{-2x}}}$

5. Evaluate  $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy$ .

6. Evaluate  $\int_1^{\infty} \frac{\tan^{-1} x}{x^2} dx$  or show that it is divergent.

7. Find the absolute maximum and minimum values of the function

$$f(x, y) = 4xy^2 - x^2y^2 - xy^3$$

on the set  $D$  the closed triangular region in  $xy$ -plane with vertices  $(0,0)$ ,  $(0,4)$  and  $(4,0)$ .

8. Find a power series representation for the function  $f(x) = x^2 \cdot \tan^{-1}(x^3)$  and determine the interval of convergence.

9. Let  $R$  be the region in the first quadrant bounded by the curve  $y = x^3$  and  $y = 2x - x^2$ . Find the volume obtained by rotating  $R$  about the  $y$ -axis.

10. Find the area of the region that lies inside the cardioid  $r = 1 + \cos \theta$  and outside the circle  $r = 3 \cos \theta$ .