編號： 282

國立成功大學 105 學年度碩士班招生考試試題

系　所：交通管理科學系

考試科目：微積分

考試日期：0228，節次：2

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

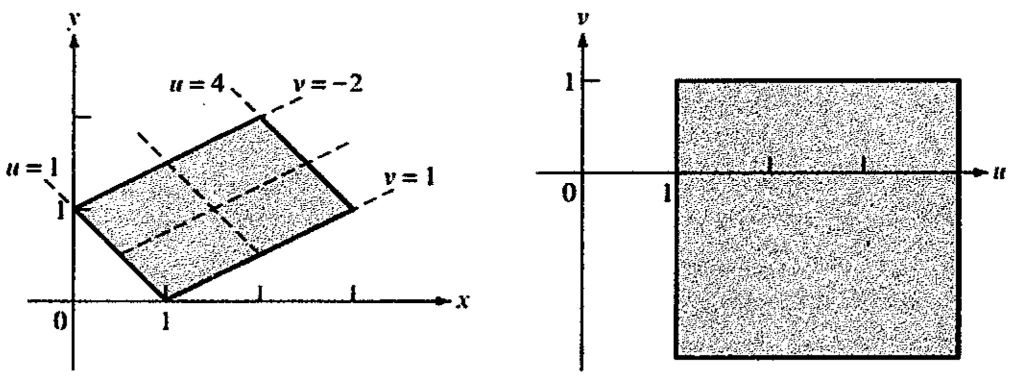
1. (20%) If $x^2+y^2-z^2=1$, then find

   (a) $\dfrac{\partial z}{\partial x}$

   (b) $\dfrac{\partial z}{\partial y}$

2. (20%) $\int_0^\infty \dfrac{1}{1+x^2}$ =

3. (20%) R_{xy} is the parallelogram shown in the following figure. The sides of R_{xy} are straight lines having equations of form $x+y=c_1$, $x-2y=c_2$. For appropriate choice of c_1, c_2. It is therefore natural to introduce as new coordinates $u=x+y$, $v=x-2y$. The region R_{xy} then corresponds to the rectangle $1\le u\le 4$, $-2\le v\le 1$. Evaluate $\iint\limits_{R_{xy}}(x+y)^3\,dxdy$.



4. (20%) Suppose that the force $\mathbf{F}$ at time t depends only on the position along the path $\mathbf{r}=\boldsymbol{\sigma}(t)$. That is, we assume that there is a vector field $\boldsymbol{\Phi}(\mathbf{r})$ such that $\mathbf{F}=\boldsymbol{\Phi}(\boldsymbol{\sigma}(t))$. The work done by the force $\mathbf{F}$ along the path $\mathbf{r}=\boldsymbol{\sigma}(t)$ is the integral $\int_{t_1}^{t_2}\mathbf{F}\cdot\dfrac{d\mathbf{r}}{dt}dt$. Find the work done by the force field $\boldsymbol{\Phi}(x,y,z)=y\mathbf{i}-x\mathbf{j}+\mathbf{k}$ as a particle is moved from $(1,0,0)$ to $(1,0,1)$ along the following path $(x,y,z)=(\cos t,\sin t,t/2\pi);0\le t\le 2\pi$.

5. (20%) $f(x)=\begin{cases}2+\sqrt{x}, & \text{if } x\ge 1\\ \dfrac{1}{2}x+\dfrac{5}{2}, & \text{if } x<1\end{cases}$. Show that f is differentiable at $x=1$.