1. What is the complementary slackness property (5%)?

2. Considering the following problem:

Maximum $Z = 2X_1 + 4X_2 + X_3$

subject to

$2X_1 \le 4$ (1)

$X_2 + X_3 \le 9$ (2)

$X_1 + X_2 \le 7$ (3)

$X_1 \ge 0, X_2 \ge 0, X_3 \ge 0$

(a) Solve this problem by simplex method (10%).

(b) In each iteration, identify the entering basic variable, leaving basic variable, obtained augmented solution, and the value of objective function (5%).

(c) List the shadow price for each resource (5%).

3. Considering the following problem:

Maximum $Z = 3X_1 + 4X_2$

subject to

$X_1 \le 5$ (1)

$X_1 - X_2 \ge 2$ (2)

$X_1 + X_2 = 4$ (3)

$X_1 \ge 0, X_2 \ge 0$

(a) Solve this problem by the revised simplex method (10%).

(b) What is the associated dual problem (5%)?

(c) Without solving the dual, identify the optimal solution in the dual problem (5%).

(d) What are the binding constraints for this problem (5%)?

4. If you need to conduct sensitivity analysis for Problem 2. You are required to independently investigate each of the following five changes in the original model. For each change, identify and evaluate the current basic solution for feasibility and for optimality. (do not reoptimize.)

(a) Change the right-hand side of constraint 2 to 5, and identify the allowable range for the right-hand side of constraint 2 (5%)

(b) Change the coefficients of X_1 to
 3 in the objective function,
 1 in constraint 1,
 2 in constraint 2, and
 3 in constraint 3 (5%)

(c) Change the coefficients of X_2 to
 3 in the objective function,
 1 in constraint 1,
 2 in constraint 2, and
 3 in constraint 3 (5%)

(d) Introduce a new constraint $2X_1 + 3X_2 \le 15$ (5%)

(e) Introduce a new variable X_6 with coefficients
 2 in the objective function,
 1 in constraint 1,
 3 in constraint 2, and
 4 in constraint 3 (5%)

(continued)

5. A company has three plants producing a certain product that is to be shipped to three distribution centers. The unit production costs are the same at the three plants, and the shipping cost (in thousands of dollars) per unit of the product is shown for each combination of plant and distribution center as follows:

	Distribution Center		
	1	2	3
Plant 1	8	5	7
Plant 2	9	4	9
Plant 3	6	8	3

The three plants can produce at most 100, 150, and 50 units per week, respectively. The three distribution centers require 100, 100, and 75 units per week, respectively. Please find the optimal shipping pattern to minimize the total shipping cost by the "Transportation Simplex Method" (15%).

6. Please find the shortest path from node 1 to node 7 for the following network, where the numbers represent actual distances between the corresponding nodes (10%).